GRAPH REWRITING AS A UNIVERSAL PROOF THEORY FOR MODERN TYPE-LOGICAL GRAMMARS

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INTRODUCTION: LANGUAGE AND LOGIC

LANGUAGE AND LOGIC

- Logic textbooks often start with an introduction relating the meaning of certain sentences (eg. "every natural number has a successor", "for all epsilon greater than zero there is a delta greater than zero such that…") to logical formulas.

THE BASIC QUESTIONS

- **Formal semantics** Can we translate all (or, at the very least, most) of natural language into first- or higher-order logic in a way which respects the meaning?
- **Type-logical grammar/categorial grammar** How can we integrate natural language syntax and semantics in a way that such a program of formal semantics can be worked out?

TRANSLATING TO LOGIC

1. every natural number has a successor

   $\forall x$ natural_number(x) $\rightarrow$ $\exists y$ successor(x,y)

2. every gambler visited a casino

   $\forall x$ gambler(x) $\rightarrow$ $\exists y$ casino(y) $\wedge$ visit(x,y)

TRANSLATING TO LOGIC

1. every natural number has a successor

   ∀x natural_number(x) → ∃y successor(x,y)

2. every <u>gambler</u> visited a casino

   ∀x <u>gambler</u>(x) → ∃y casino(y) ∧ visit(x,y)

TRANSLATING TO LOGIC

1. every natural number has a successor

   $\forall x$ natural_number(x) → $\exists y$ successor(x,y)

2. every gambler <u>visited</u> a casino

   $\forall x$ gambler(x) → $\exists y$ casino(y) ∧ <u>visit</u>(x,y)

TRANSLATING TO LOGIC

1. every natural number has a successor

   $\forall x\ natural\_number(x) \rightarrow \exists y\ successor(x,y)$

2. every gambler visited a <u>casino</u>

   $\forall x\ gambler(x) \rightarrow \exists y\ \underline{casino}(y) \wedge visit(x,y)$

TRANSLATING TO LOGIC

1. every natural number has a successor

   ∀x natural_number(x) → ∃y successor(x,y)

2. <u>every</u> gambler visited a casino

   <u>∀x</u> gambler(x) <u>→</u> ∃y casino(y) ∧ visit(x,y)

TRANSLATING TO LOGIC

1. every natural number has a successor

   $\forall x \; natural\_number(x) \rightarrow \exists y \; successor(x,y)$

2. every gambler visited <u>a</u> casino

   $\forall x \; gambler(x) \rightarrow \underline{\exists y} \; casino(y) \; \underline{\wedge} \; visit(x,y)$

TRANSLATING TO LOGIC

- Many of the words corresponding (at least more or less) to the standard logical connectives "no", "all", "some" but also "didn't" seem to have a sort of mismatch between the natural language sentence and the corresponding formula
- How can we "fix" this mismatch?

TRANSLATING TO LOGIC

- How can we “fix” this mismatch?

Joachim Lambek
(1922-2014)

Richard Montague
(1930-1971)

THE LAMBEK CALCULUS

$$\frac{A/B \quad B}{A}\ /E \qquad \frac{B \quad B\backslash A}{A}\ \backslash E$$

$$\begin{array}{c} \ldots \quad [B]^i \\ \vdots \\ \dfrac{A}{A/B}\ /I \end{array} \qquad \begin{array}{c} [B]^i \quad \ldots \\ \vdots \\ \dfrac{A}{B\backslash A}\ \backslash I \end{array}$$

THE LAMBEK CALCULUS

$$\frac{A/B \quad B}{A}\ /E \qquad \frac{B \quad B\backslash A}{A}\ \backslash E$$

$$\cfrac{\cfrac{\cfrac{\text{the}}{np/n}\ Lex \quad \cfrac{\text{student}}{n}\ Lex}{np}\ /E \quad \cfrac{\text{slept}}{np\backslash s}\ Lex}{s}\ \backslash E$$

EVERY GAMBLER VISITED A CASINO

$$\frac{\text{every}}{(s/(np\backslash s))/n} \quad \frac{\text{gambler}}{n} \quad \frac{\text{visited}}{(np\backslash s)/np} \quad \frac{\text{a}}{((s/np)\backslash s)/n} \quad \frac{\text{casino}}{n}$$

EVERY GAMBLER VISITED A CASINO

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n} \quad \dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E \qquad \dfrac{\text{visited}}{(np\backslash s)/np} \qquad \dfrac{\text{a}}{((s/np)\backslash s)/n} \qquad \dfrac{\text{casino}}{n}$$

EVERY GAMBLER VISITED A CASINO

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n} \qquad \dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\ /E \qquad \dfrac{\text{visited}}{(np\backslash s)/np} \qquad \dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n} \qquad \dfrac{\text{casino}}{n}}{(s/np)\backslash s}\ /E$$

EVERY GAMBLER VISITED A CASINO

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n} \qquad \dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\,/E \qquad \dfrac{\text{visited}}{(np\backslash s)/np} \qquad np \qquad \dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n} \qquad \dfrac{\text{casino}}{n}}{(s/np)\backslash s}\,/E$$

EVERY GAMBLER VISITED A CASINO

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n} \qquad \dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\ /E \qquad \dfrac{\dfrac{\text{visited}}{(np\backslash s)/np} \qquad np}{np\backslash s}\ /E \qquad \dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n} \qquad \dfrac{\text{casino}}{n}}{(s/np)\backslash s}\ /E$$

EVERY GAMBLER VISITED A CASINO

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n} \qquad \dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\ /E \qquad np \qquad \dfrac{\dfrac{\text{visited}}{(np\backslash s)/np} \quad np}{np\backslash s}\ /E \qquad \dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n} \qquad \dfrac{\text{casino}}{n}}{(s/np)\backslash s}\ /E$$

EVERY GAMBLER VISITED A CASINO

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n} \quad \dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\ /E \qquad \dfrac{np \quad \dfrac{\dfrac{\text{visited}}{(np\backslash s)/np} \quad np}{np\backslash s}\ /E}{s}\ \backslash E \qquad \dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n} \quad \dfrac{\text{casino}}{n}}{(s/np)\backslash s}\ /E$$

EVERY GAMBLER VISITED A CASINO

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n} \quad \dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\ /E$$

$$\dfrac{\dfrac{np \quad \dfrac{\dfrac{\text{visited}}{(np\backslash s)/np} \quad [np]^1}{np\backslash s}\ /E}{s}\ \backslash E}{s/np}\ /I_1$$

$$\dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n} \quad \dfrac{\text{casino}}{n}}{(s/np)\backslash s}\ /E$$

EVERY GAMBLER VISITED A CASINO

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n} \qquad \dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\ /E$$

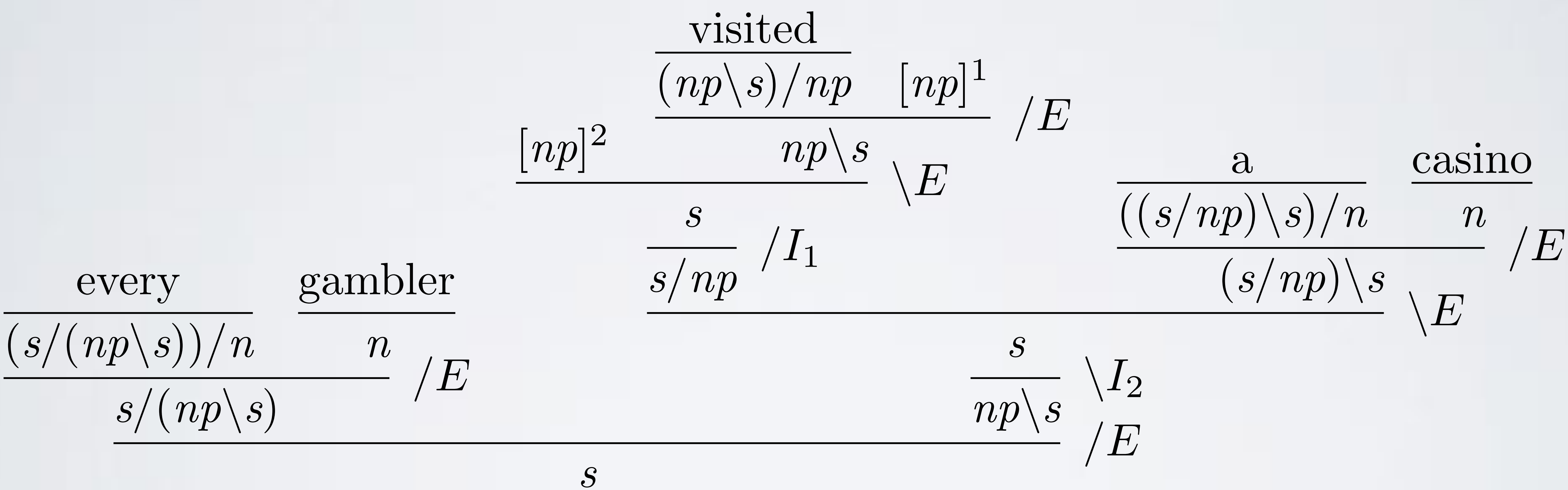
$$\dfrac{\dfrac{\dfrac{np \qquad \dfrac{\dfrac{\text{visited}}{(np\backslash s)/np} \quad [np]^1}{np\backslash s}\ /E}{s}\ \backslash E}{s/np}\ /I_1 \qquad \dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n} \quad \dfrac{\text{casino}}{n}}{(s/np)\backslash s}\ /E}{s}\ \backslash E$$

EVERY GAMBLER VISITED A CASINO

$$\dfrac{\dfrac{\text{every}}{(s/(np\backslash s))/n} \quad \dfrac{\text{gambler}}{n}}{s/(np\backslash s)}\ /E$$

$$\dfrac{\dfrac{\dfrac{\dfrac{[np]^2 \quad \dfrac{\dfrac{\text{visited}}{(np\backslash s)/np} \quad [np]^1}{np\backslash s}\ /E}{s}\ \backslash E}{s/np}\ /I_1 \quad \dfrac{\dfrac{\text{a}}{((s/np)\backslash s)/n} \quad \dfrac{\text{casino}}{n}}{(s/np)\backslash s}\ /E}{s}\ \backslash E}{np\backslash s}\ \backslash I_2$$

EVERY GAMBLER VISITED A CASINO



EVERY GAMBLER VISITED A CASINO "DEEP STRUCTURE"

$$\dfrac{\dfrac{n \multimap ((np \multimap s) \multimap s) \quad n}{(np \multimap s) \multimap s}\multimap E \qquad \dfrac{\dfrac{\dfrac{\dfrac{[np]^2 \quad \dfrac{np \multimap (np \multimap s) \quad [np]^1}{np \multimap s}\multimap E}{s}\multimap E}{np \multimap s}\multimap I_1 \quad \dfrac{n \multimap ((np \multimap s) \multimap s) \quad n}{(np \multimap s) \multimap s}\multimap E}{s}\multimap E}{np \multimap s}\multimap I_2}{s}\multimap E$$

The Lambek calculus is the intuitionistic, multiplicative, non-commutative fragment of linear logic. If we replace "/" and "\\" by "$\multimap$" we obtain a linear logic proof.

SYNTACTIC TYPES TO SEMANTIC TYPES

$$
\begin{aligned}
np^* &= e \\
n^* &= e \rightarrow t \\
s^* &= t \\
(A \multimap B)^* &= A^* \rightarrow B^*
\end{aligned}
$$

$$
\begin{aligned}
(np \multimap (np \multimap s))* &= e \rightarrow (e \rightarrow t) \\
(np \multimap s) \multimap s)* &= (e \rightarrow t) \rightarrow t \\
(n \multimap (np \multimap s) \multimap s))* &= (e \rightarrow t) \rightarrow ((e \rightarrow t) \rightarrow t)
\end{aligned}
$$

SEMANTIC DERIVATION AND LAMBDA TERM

$$
\dfrac{\dfrac{z_0^{(e\to t)\to(e\to t)\to t} \qquad z_1^{e\to t}}{(z_0\, z_1)^{(e\to t)\to t}}\to E
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[x^e]^2 \qquad \dfrac{z_2^{e\to(e\to t)} \qquad [y^e]^1}{(z_2\, y)^{e\to t}}\to E}{((z_2\, y)\, x)^t}\to E}{\lambda y.((z_2\, y)\, x)^{e\to t}}\to I_1
\qquad
\dfrac{z_3^{(e\to t)\to(e\to t)\to t} \qquad z_4^{e\to t}}{(z_3\, z_4)^{(e\to t)\to t}}\to E}{((z_3\, z_4)\, \lambda y.((z_2\, y)\, x))^t}\to E}{\lambda x.((z_3\, z_4)\, \lambda y.((z_2\, y)\, x))^{e\to t}}\to I_2}{((z_0\, z_1)\, (\lambda x.((z_3\, z_4)\, \lambda y.((z_2\, y)\, x))))^t}\to E
$$

THE LEXICAL MEANING OF "EVERY"

$$(n \multimap (np \multimap s) \multimap s))* = (e \to t) \to ((e \to t) \to t)$$

$$\lambda P^{e \to t}.\lambda Q^{e \to t}.(\forall^{(e \to t) \to t}(\lambda x^{e}.((\Rightarrow^{t \to (t \to t)} (P\, x))(Q\, x))))$$

THE LEXICAL MEANING OF "EVERY"

$$\lambda P^{e \to t}.\lambda Q^{e \to t}.(\forall^{(e \to t) \to t}(\lambda x^{e}.((\Rightarrow^{t \to (t \to t)}\ (P\,x))(Q\,x))))$$

$$\lambda P^{e \to t}.\lambda Q^{e \to t}.\forall x^{e}.[(P\,x) \Rightarrow (Q\,x)]$$

THE LEXICAL MEANING OF "EVERY"

$$\lambda P^{e\to t}.\lambda Q^{e\to t}.(\forall^{(e\to t)\to t}(\lambda x^{e}.((\Rightarrow^{t\to(t\to t)}\ (P\,x))(Q\,x))))$$

$$\lambda P^{e\to t}.\lambda Q^{e\to t}.\forall x^{e}.[(P\,x)\Rightarrow(Q\,x)]$$

$$\lambda P^{e\to t}.\lambda Q^{e\to t}.(P\ \subseteq\ Q)$$

THE LEXICAL MEANING OF "A"

$$\lambda P^{e\to t}.\lambda Q^{e\to t}.(\exists^{(e\to t)\to t}(\lambda x^e.((\wedge^{t\to(t\to t)}(P\,x))(Q\,x))))$$

$$\lambda P^{e\to t}.\lambda Q^{e\to t}.\exists x^e.[(P\,x)\wedge(Q\,x)]$$

$$\lambda P^{e\to t}.\lambda Q^{e\to t}.(P\cap Q)\neq\emptyset$$

LEXICAL SUBSTITUTION

$$((z_0\, z_1)\, (\lambda x.((z_3\, z_4)\, \lambda y.((z_2\, y)\, x))))$$

$$z_0 := \lambda P^{e\to t}.\lambda Q^{e\to t}.(\forall(\lambda x^e.((\Rightarrow\, (P\, x))(Q\, x))))$$

$$z_1 := gambler^{e\to t}$$

$$z_2 := visit^{e\to(e\to t)}$$

$$z_3 := \lambda P^{e\to t}.\lambda Q^{e\to t}.(\exists(\lambda x^e.((\wedge(P\, x))(Q\, x))))$$

$$z_4 := casino^{e\to t}$$

LEXICAL SUBSTITUTION

$$((\lambda P^{e\to t}.\lambda Q^{e\to t}.(\forall(\lambda v^{e}.((\Rightarrow (P\,v))(Q\,v))))\ gambler^{e\to t})$$

$$(\lambda x.((\lambda P'^{e\to t}.\lambda Q'^{e\to t}.(\exists(\lambda z^{e}.((\wedge(P'\,z))(Q'\,z))))\ casino^{e\to t})$$

$$\lambda y.((visit^{e\to(e\to t)}\ y)\ x))))$$

NORMALISATION

$$((\lambda P^{e\to t}.\lambda Q^{e\to t}.(\forall(\lambda v^{e}.((\Rightarrow (P\,v))(Q\,v))))\ gambler^{e\to t})$$
$$(\lambda x.((\lambda P'^{e\to t}.\lambda Q'^{e\to t}.(\exists(\lambda z^{e}.((\wedge(P'\,z))(Q'\,z))))\ casino^{e\to t})$$
$$\lambda y.((visit^{e\to(e\to t)}\ y)\ x))))$$

$$\rightsquigarrow_{\beta} (\forall(\lambda x^{e}.((\Rightarrow (gambler^{e\to t}\ x))(\exists(\lambda y^{e}.((\wedge(casino^{e\to t}\ y))((visit^{e\to(e\to t)}\ y)\ x)))))))$$

NORMALISATION

$$((\lambda P^{e\to t}.\lambda Q^{e\to t}.(\forall(\lambda v^{e}.((\Rightarrow (P\, v))(Q\, v)))))\ gambler^{e\to t})$$
$$(\lambda x.((\lambda P'^{e\to t}.\lambda Q'^{e\to t}.(\exists(\lambda z^{e}.((\wedge(P'\, z))(Q'\, z)))))\ casino^{e\to t})$$
$$\lambda y.((visit^{e\to(e\to t)}\ y)\ x))))$$

$$\leadsto_{\beta} (\forall(\lambda x^{e}.((\Rightarrow (gambler^{e\to t}\ x))(\exists(\lambda y^{e}.((\wedge(casino^{e\to t}\ y))((visit^{e\to(e\to t)}\ y)\ x)))))))$$

$$\equiv_{def} \forall x.[gambler(x) \Rightarrow \exists y.[casino(y) \wedge visit(x,y)]]$$

LAMBEK AND MONTAGUE

- Montague’s strategy makes the apparent mismatch between syntax and semantics disappear.
- Syntax and semantics are developed in parallel.

PROBLEMS AND EXTENSIONS

- Most variants and extensions of the Lambek calculus agree on the “deep structure”, the (multiplicative, intuitionistic) linear logic proof used for the computation of semantics.

- However, the “surface structure” of these logics are rather different: different connectives, structures, operations…

DE DICTO/DE RE

"John believes someone left"

$$
\dfrac{\dfrac{\text{John}}{np}\,Lex \qquad \dfrac{\dfrac{\text{believes}}{s \multimap (np \multimap s)}\,Lex \qquad \dfrac{\dfrac{\text{someone}}{(np \multimap s) \multimap s}\,Lex \qquad \dfrac{\text{left}}{np \multimap s}\,Lex}{s}\multimap E}{np \multimap s}\multimap E}{s}\multimap E
$$

DE DICTO/DE RE

"John believes someone left"

$$
\dfrac{\dfrac{\text{someone}}{(np \multimap s) \multimap s}\,Lex \qquad \dfrac{\dfrac{\dfrac{\text{John}}{np}\,Lex \qquad \dfrac{\dfrac{\text{believes}}{s \multimap (np \multimap s)}\,Lex \qquad \dfrac{[np]^1 \qquad \dfrac{\text{left}}{np \multimap s}\,Lex}{s}\multimap E}{np \multimap s}\multimap E}{s}\multimap E}{np \multimap s}\multimap I_1}{s}\multimap E
$$

This is not the forgetful mapping of *any* Lambek calculus proof!

(at least not given np\s for "left" and (np\s)/s for "believes")

DUTCH VERB CLUSTERS

"(dat Jan) Henk Marie de nijlpaarden zag helpen voeren"

$$\dfrac{\dfrac{\dfrac{\textrm{zag}}{np \multimap (inf \multimap (np \multimap s))}\,Lex \quad \dfrac{\textrm{Henk}}{np}\,Lex}{inf \multimap (np \multimap s)}\multimap E \qquad \dfrac{\dfrac{\dfrac{\textrm{Marie}}{np}\,Lex \quad \dfrac{\textrm{helpen}}{np \multimap (inf \multimap inf)}\,Lex}{inf \multimap inf}\multimap E \quad \dfrac{\dfrac{\textrm{de nijlpaarden}}{np}\,Lex \quad \dfrac{\textrm{voeren}}{np \multimap inf}\,Lex}{inf}\multimap E}{inf}\multimap E}{np \multimap s}\multimap E$$

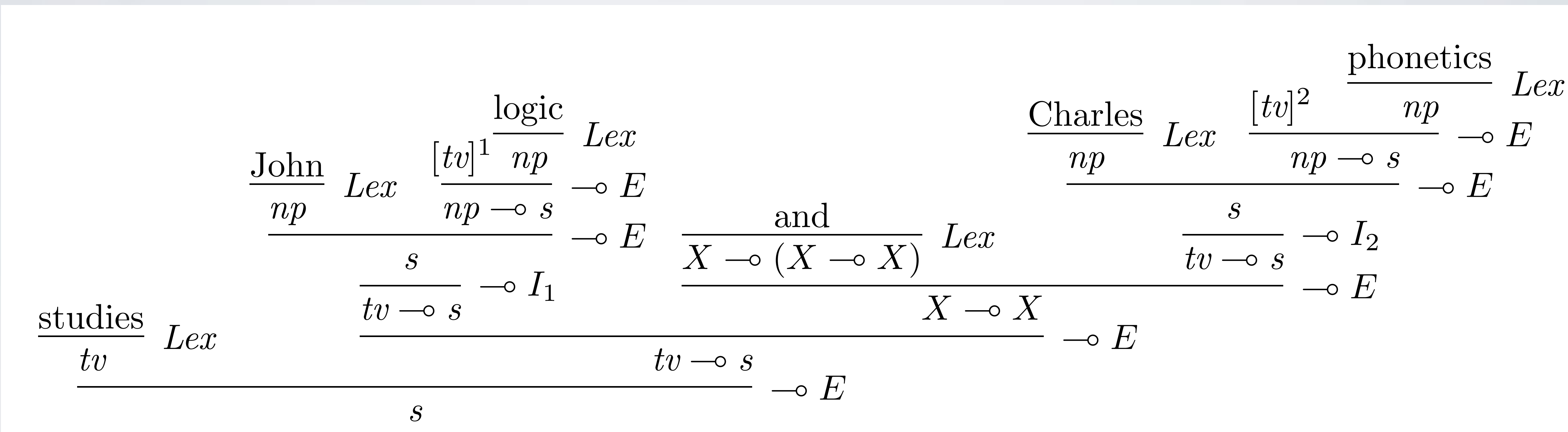
DUTCH VERB CLUSTERS

"(dat Jan) Henk Marie de nijlpaarden zag helpen voeren"

$$
\dfrac{
  \dfrac{\text{Henk}}{np}\,Lex
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{\text{Marie}}{np}\,Lex
      \qquad
      \dfrac{
        \dfrac{
          \dfrac{\text{de nijlpaarden}}{np}\,Lex
          \qquad
          \dfrac{\text{zag}}{np\backslash(np\backslash s)}\,Lex
        }{np\backslash s}\,\backslash E
        \qquad
        \dfrac{\text{helpen}}{(np\backslash s)\backslash(np\backslash(np\backslash s))}\,Lex
      }{np\backslash(np\backslash s)}\,\backslash E
    }{np\backslash s}\,\backslash E
    \qquad
    \dfrac{\text{voeren}}{(np\backslash s)\backslash(np\backslash(np\backslash s))}\,Lex
  }{np\backslash(np\backslash s)}\,\backslash E
}{np\backslash s}\,\backslash E
$$

GAPPING

"John studies logic and Charles phonetics"



$$tv = np \multimap np \multimap s$$

$$X = tv \multimap s$$

$$= (np \multimap np \multimap s) \multimap s$$

VP ELLIPSIS

"John left before Mary did"

$$
\dfrac{\dfrac{\text{John}}{np}\,Lex \qquad \dfrac{\dfrac{\text{left}}{vp}\,Lex \qquad \dfrac{\dfrac{\dfrac{\dfrac{[vp]^1 \qquad \dfrac{\dfrac{\text{before}}{s \multimap (vp \multimap vp)}\,Lex \qquad \dfrac{\dfrac{\text{Mary}}{np}\,Lex \qquad [vp]^2}{s}\,{\multimap}E}{vp \multimap vp}\,{\multimap}E}{vp}\,{\multimap}E}{vp \multimap vp}\,{\multimap}I_1}{vp \multimap (vp \multimap vp)}\,{\multimap}I_2 \qquad \dfrac{\text{did}}{(vp \multimap (vp \multimap vp)) \multimap (vp \multimap vp)}\,Lex}{vp \multimap vp}\,{\multimap}E}{np \multimap s}\,{\multimap}E}{s}\,{\multimap}E
$$

$$vp = np \multimap s$$

EXTENDING THE LAMBEK CALCULUS

- Grammar design in type-logical grammars can be viewed as a form of "reverse engineering" based on a semantic structure (i.e. a linear logic proof).
- Lambek grammars have only the option of choosing a direction for the slashes; other systems allow discontinuous dependencies.

GOING FURTHER

- The Lambek calculus gives a simple account of some elementary facts about the syntax-semantics interface.
- However, once we want to handle more complex examples, we run into problems.
- Many variants and extensions of the Lambek calculus have been proposed to solve these problems.

MODERN TYPE-LOGICAL GRAMMARS

MODERN TYPE-LOGICAL GRAMMARS

- We are looking for a logic which solves the problems with the Lambek calculus, while not sacrificing simplicity and good logical properties.
- Many solutions have been proposed, which makes comparisons different.
- There is a "family resemblance" between many of the proposed analyses, but can we make this more precise?

MULTIMODAL

$$\frac{\Delta \vdash A \bullet_i B \quad \Gamma[(A \circ_i B)] \vdash C}{\Gamma[\Delta] \vdash C} [\bullet E] \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{(\Gamma \circ_i \Delta) \vdash A \bullet_i B} [\bullet I]$$

$$\frac{\Gamma \vdash A/_i B \quad \Delta \vdash B}{(\Gamma \circ_i \Delta) \vdash A} [/E] \qquad \frac{(\Gamma \circ_i B) \vdash A}{\Gamma \vdash A/_i B} [/I]$$

$$\frac{\Gamma \vdash B \quad \Delta \vdash B\backslash_i A}{(\Gamma \circ_i \Delta) \vdash A} [\backslash E] \qquad \frac{(B \circ_i \Gamma) \vdash A}{\Gamma \vdash B\backslash_i A} [\backslash I]$$

Oehrle & Zhang (1989), Moortgat & Morrill (1991), Moortgat & Oehrle (1993,1994), Hepple (1994)

MULTIMODAL

$$\frac{\Gamma[\Delta_1 \circ_2 (\Delta_2 \circ_1 \Delta_3)] \vdash C}{\Gamma[(\Delta_1 \circ_2 \Delta_2) \circ_1 \Delta_3] \vdash C}\ MA \qquad \frac{\Gamma[\Delta_2 \circ_2 (\Delta_1 \circ_1 \Delta_3)] \vdash C}{\Gamma[\Delta_1 \circ_1 (\Delta_2 \circ_2 \Delta_3)] \vdash C}\ MC$$

$$\cfrac{\mathrm{Marie} \vdash np \qquad \cfrac{\mathrm{wil} \vdash (np\backslash_1 s)/_1 inf \qquad \cfrac{\mathrm{boeken} \vdash np \quad \mathrm{lezen} \vdash np\backslash_2 inf}{\mathrm{boeken} \circ_2 \mathrm{lezen} \vdash inf}\ \backslash E}{\mathrm{wil} \circ_1 (\mathrm{boeken} \circ_2 \mathrm{lezen}) \vdash np\backslash s}\ /E}{\cfrac{\mathrm{Marie} \circ_1 (\mathrm{wil} \circ_1 (\mathrm{boeken} \circ_2 \mathrm{lezen})) \vdash s}{\mathrm{Marie} \circ_1 (\mathrm{boeken} \circ_2 (\mathrm{wil} \circ_1 \mathrm{lezen})) \vdash s}\ MC}\ \backslash E$$

For details, see Moortgat & Oehrle (1994), Oehrle (2011)

DISPLACEMENT CALCULUS

$$\dfrac{\delta : A \odot B \qquad \begin{array}{c}[\alpha_1 + \mathbf{1} + \alpha_2 : A]^i \quad [\beta : B]^i \\ \vdots \\ \gamma_1 + \alpha_1 + \beta + \alpha_2 + \gamma_2 : C\end{array}}{\gamma_1 + \delta + \gamma_2 : C} \odot E_i$$

$$\dfrac{\alpha_1 + \mathbf{1} + \alpha_2 : A \quad \beta : B}{\alpha_1 + \beta + \alpha_2 : A \odot B} \odot I$$

$$\dfrac{\alpha_1 + \mathbf{1} + \alpha_2 : C \uparrow B \quad \beta : B}{\alpha_1 + \beta + \alpha_2 : C} \uparrow E$$

$$\dfrac{\begin{array}{c}[\beta : B]^i \\ \vdots \\ \alpha_1 + \beta + \alpha_2 : C\end{array}}{\alpha_1 + \mathbf{1} + \alpha_2 : C \uparrow B} \uparrow I_i$$

$$\dfrac{\alpha_1 + \mathbf{1} + \alpha_2 : A \quad \beta : A \downarrow C}{\alpha_1 + \beta + \alpha_2 : C} \downarrow E$$

$$\dfrac{\begin{array}{c}[\alpha_1 + \mathbf{1} + \alpha_2 : A]^i \\ \vdots \\ \alpha_1 + \beta + \alpha_2 : C\end{array}}{\beta : A \downarrow C} \downarrow I_i$$

DISPLACEMENT CALCULUS

$$\dfrac{\text{someone} : (s \uparrow np) \downarrow s \quad \dfrac{\begin{array}{c} a : np \\ \vdots \\ \text{Mary} + \text{thinks} + a + \text{left} : s \end{array}}{\text{Mary} + \text{thinks} + \mathbf{1} + \text{left} : s \uparrow np} \uparrow I}{\text{Mary} + \text{thinks} + \text{someone} + \text{left} : s} \downarrow E$$

Morrill, Valentin & Fadda (2011)

HYBRID TYPE-LOGICAL GRAMMARS

$$\frac{\Gamma \vdash M : A \multimap B \quad \Delta \vdash N : A}{\Gamma, \Delta \vdash (M\ N) : B} \multimap E \qquad \frac{\Gamma, x : A \vdash M : B}{\Gamma \vdash \lambda x.M : A \multimap B} \multimap I$$

$$\frac{\Gamma \vdash P : B \quad \Delta \vdash Q : B\backslash A}{\Gamma, \Delta \vdash P + Q : A} \backslash E \qquad \frac{w : B, \Gamma \vdash w + P : A}{\Gamma \vdash P : B\backslash A} \backslash I$$

$$\frac{\Gamma \vdash P : A/B \quad \Delta \vdash Q : B}{\Gamma, \Delta \vdash P + Q : A} /E \qquad \frac{\Gamma, w : B \vdash P + w : A}{\Gamma \vdash P : A/B} /I$$

HTLG: GAPPING

$$\dfrac{\vdots}{Charles + y + phonetics : s}$$

$$\dfrac{Charles + y + phonetics : s}{\lambda y.Charles + y + phonetics : tv \multimap s} \multimap I$$

$$\dfrac{\vdots}{John + x + logic : s}$$

$$\dfrac{John + x + logic : s}{\lambda x.John + x + logic : tv \multimap s}$$

$$\dfrac{\lambda Q \lambda P \lambda v.(P\ v) + and + (Q\ \epsilon) : X \multimap X \multimap X \quad \lambda y.Charles + y + phonetics : tv \multimap s}{\lambda P \lambda v.and + Charles + phonetics : X \multimap X} \multimap E$$

$$\dfrac{\lambda x.John + x + logic : tv \multimap s \quad \lambda P \lambda v.and + Charles + phonetics : X \multimap X}{\lambda p.John + p + logic + and + Charles + phonetics : tv \multimap s} \multimap E$$

$$\dfrac{studies \vdash tv \quad \lambda p.John + p + logic + and + Charles + phonetics : tv \multimap s}{John + studies + logic + and + Charles + phonetics : s} \multimap E$$

Kubota & Levine (2012, 2020)

THE LOGIC OF SCOPE

NLλ

$$\frac{\Gamma \vdash C \,/\, B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash C}\ /E \qquad \frac{(\Gamma \circ B) \vdash C}{\Gamma \vdash C \,/\, B}\ /I$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash A \,\backslash\, C}{\Gamma \circ \Delta \vdash C}\ \backslash E \qquad \frac{(A \circ \Gamma) \vdash C}{\Gamma \vdash A \,\backslash\, C}\ \backslash I$$

$$\frac{\Gamma[(A \circ B)] \vdash D}{\Gamma[A \bullet B] \vdash D}\ \bullet E \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{(\Gamma \circ \Delta) \vdash A \bullet B}\ \bullet I$$

$$\frac{\Gamma \vdash C \mathbin{/\!\!/} B \quad \Delta \vdash B}{\Gamma \circledcirc \Delta \vdash C}\ /E \qquad \frac{(\Gamma \circledcirc B) \vdash C}{\Gamma \vdash C \mathbin{/\!\!/} B}\ /I$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash A \mathbin{\backslash\!\!\backslash} C}{\Gamma \circledcirc \Delta \vdash C}\ \backslash E \qquad \frac{(A \circledcirc \Gamma) \vdash C}{\Gamma \vdash A \mathbin{\backslash\!\!\backslash} C}\ \backslash I$$

$$\frac{\Gamma[(A \circledcirc B)] \vdash D}{\Gamma[A \odot B] \vdash D}\ \bullet E \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{(\Gamma \circledcirc \Delta) \vdash A \odot B}\ \bullet I$$

$$\frac{\Xi[(\Delta \circledcirc \lambda x.\Gamma[x])] \vdash D}{\Xi[\Gamma[\Delta]] \vdash D}\ \beta \qquad \frac{\Xi[\Gamma[\Delta]] \vdash D}{\Xi[(\Delta \circledcirc \lambda x.\Gamma[x])] \vdash D}\ \beta^{-1}$$

THE LOGIC OF SCOPE

NLλ

$$\frac{\Gamma \vdash C\,/\,B \quad \Delta \vdash B}{\Gamma \circ \Delta \vdash C}\ /E \qquad \frac{(\Gamma \circ B) \vdash C}{\Gamma \vdash C\,/\,B}\ /I$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash A \backslash C}{\Gamma \circ \Delta \vdash C}\ \backslash E \qquad \frac{(A \circ \Gamma) \vdash C}{\Gamma \vdash A \backslash C}\ \backslash I$$

$$\frac{\Gamma[(A \circ B)] \vdash D}{\Gamma[A \bullet B] \vdash D}\ \bullet E \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{(\Gamma \circ \Delta) \vdash A \bullet B}\ \bullet I$$

THE LOGIC OF SCOPE

NLλ

$$\frac{\Gamma \vdash C /\!\!/ B \quad \Delta \vdash B}{\Gamma \circledcirc \Delta \vdash C} \; /E \qquad \frac{(\Gamma \circledcirc B) \vdash C}{\Gamma \vdash C /\!\!/ B} \; /I$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash A \backslash\!\!\backslash C}{\Gamma \circledcirc \Delta \vdash C} \; \backslash E \qquad \frac{(A \circledcirc \Gamma) \vdash C}{\Gamma \vdash A \backslash\!\!\backslash C} \; \backslash I$$

$$\frac{\Gamma[(A \circledcirc B)] \vdash D}{\Gamma[A \odot B] \vdash D} \; \bullet E \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{(\Gamma \circledcirc \Delta) \vdash A \odot B} \; \bullet I$$

THE LOGIC OF SCOPE

NLλ

$$\frac{\Gamma \vdash C \mathbin{/\!\!/} B \quad \Delta \vdash B}{\Gamma \circledcirc \Delta \vdash C} \; /E \qquad \frac{(\Gamma \circledcirc B) \vdash C}{\Gamma \vdash C \mathbin{/\!\!/} B} \; /I$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash A \mathbin{\backslash\!\!\backslash} C}{\Gamma \circledcirc \Delta \vdash C} \; \backslash E \qquad \frac{(A \circledcirc \Gamma) \vdash C}{\Gamma \vdash A \mathbin{\backslash\!\!\backslash} C} \; \backslash I$$

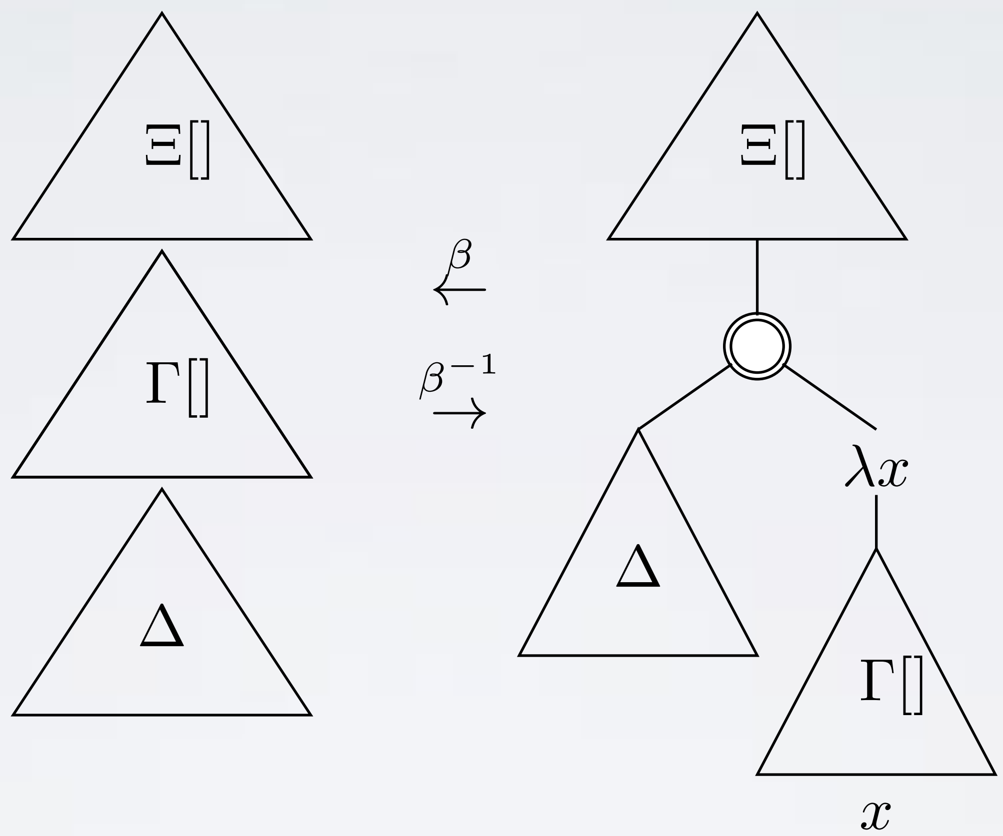
$$\frac{\Gamma[(A \circledcirc B)] \vdash D}{\Gamma[A \odot B] \vdash D} \; \bullet E \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{(\Gamma \circledcirc \Delta) \vdash A \odot B} \; \bullet I$$

$$\frac{\Xi[(\Delta \circledcirc \lambda x.\Gamma[x])] \vdash D}{\Xi[\Gamma[\Delta]] \vdash D} \; \beta \qquad \frac{\Xi[\Gamma[\Delta]] \vdash D}{\Xi[(\Delta \circledcirc \lambda x.\Gamma[x])] \vdash D} \; \beta^{-1}$$

Barker & Shan (2014), Barker (2019)

THE LOGIC OF SCOPE

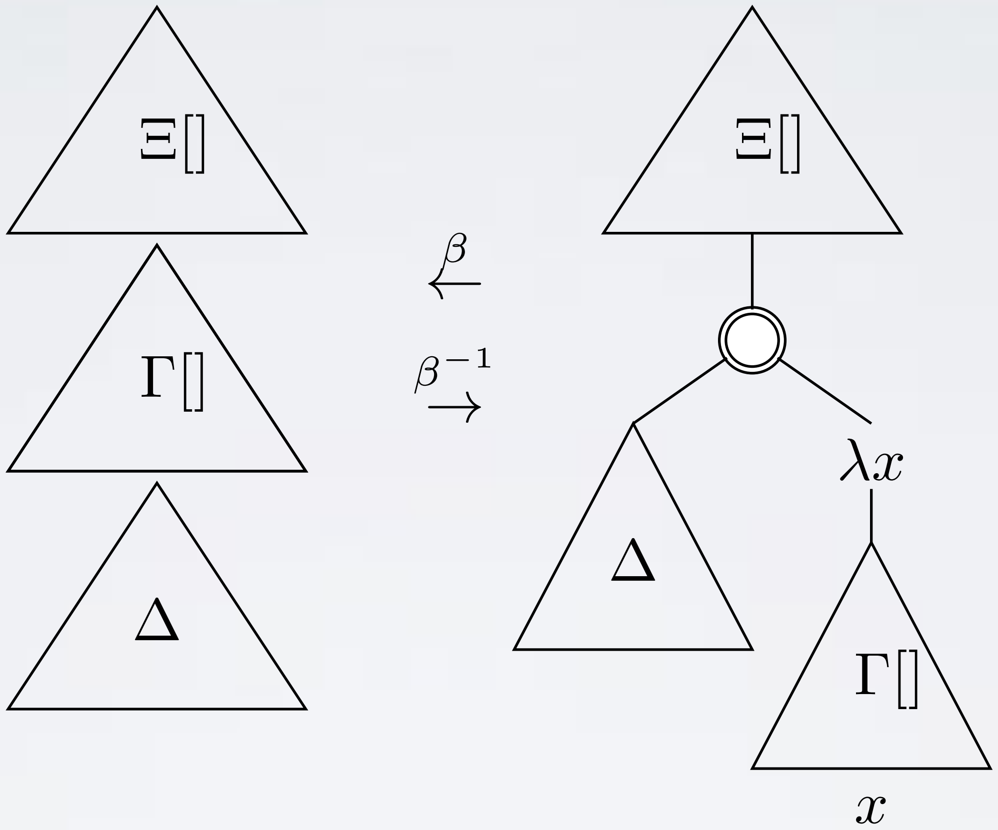
NLλ



$$\frac{\Xi[(\Delta \circledcirc \lambda x.\Gamma[x])] \vdash D}{\Xi[\Gamma[\Delta]] \vdash D}\ \beta \qquad \frac{\Xi[\Gamma[\Delta]] \vdash D}{\Xi[(\Delta \circledcirc \lambda x.\Gamma[x])] \vdash D}\ \beta^{-1}$$

THE LOGIC OF SCOPE

NLλ



$$(\lambda x.M)N \equiv M[x ::= N]$$

$$\frac{\Xi[(\Delta \circledcirc \lambda x.\Gamma[x])] \vdash D}{\Xi[\Gamma[\Delta]] \vdash D}\ \beta \qquad \frac{\Xi[\Gamma[\Delta]] \vdash D}{\Xi[(\Delta \circledcirc \lambda x.\Gamma[x])] \vdash D}\ \beta^{-1}$$

THE LOGIC OF SCOPE

NLλ

$$
\dfrac{\dfrac{everyone \vdash s \mathbin{/\!\!/} (np \mathbin{\backslash\!\!\backslash} s) \qquad \dfrac{\dfrac{\dfrac{\vdots}{John \circ (saw \circ np) \vdash s}}{np \circledcirc \lambda x.(John \circ (saw \circ x)) \vdash s}\,\beta^{-1}}{\lambda x.(John \circ (saw \circ x)) \vdash np \mathbin{\backslash\!\!\backslash} s}\,\backslash\!\!\backslash I}{everyone \circledcirc \lambda x.(John \circ (saw \circ x)) \vdash s}\,/\!\!/E}{John \circ (saw \circ everyone) \vdash s}\,\beta
$$

$$
\dfrac{\Xi[(\Delta \circledcirc \lambda x.\Gamma[x])] \vdash D}{\Xi[\Gamma[\Delta]] \vdash D}\,\beta \qquad\qquad \dfrac{\Xi[\Gamma[\Delta]] \vdash D}{\Xi[(\Delta \circledcirc \lambda x.\Gamma[x])] \vdash D}\,\beta^{-1}
$$

MODERN TYPE-LOGICAL GRAMMARS

Logic	Connectives	Structure	Operations
L	$/, \bullet, \backslash$	list	—
NL	$/, \bullet, \backslash$	binary tree	—
Multimodal	$/_i, \bullet_i, \backslash_i$	labeled binary tree	tree rewrites
	$\Diamond_j, \Box_j$	labeled 1-2 tree	tree rewrites
D	$\mathbf{L} + \uparrow_k, \odot_k, \downarrow_k$		
	${}^\wedge, {}^\vee$	tuple of lists	wrap
Lambda	$\multimap$	lambda term	β reduction
Hybrid	$\mathbf{L} + \multimap$	lambda term (list)	β reduction
$\mathbf{NL}_\lambda$	$\mathbf{NL} + /\!\!/, \circledcirc, \backslash\!\!\backslash$	lambda term (tree)	β reduction/expansion
LG	$\mathbf{NL} + \oslash, \circledast, \obslash$	free tree	graph rewrites

PROOF NETS

PROOF NETS

- Optimal (redundancy-free) representation of proofs in multiplicative linear logic (Girard 1987)
- Adapted to the Lambek calculus (Roorda 1991) and to multimodal categorial grammars (Moot & Puite 2002)
- What about other modern type-logical grammars?

PROOF NETS AND PROOF SEARCH

- Proof search for proof nets is very easy

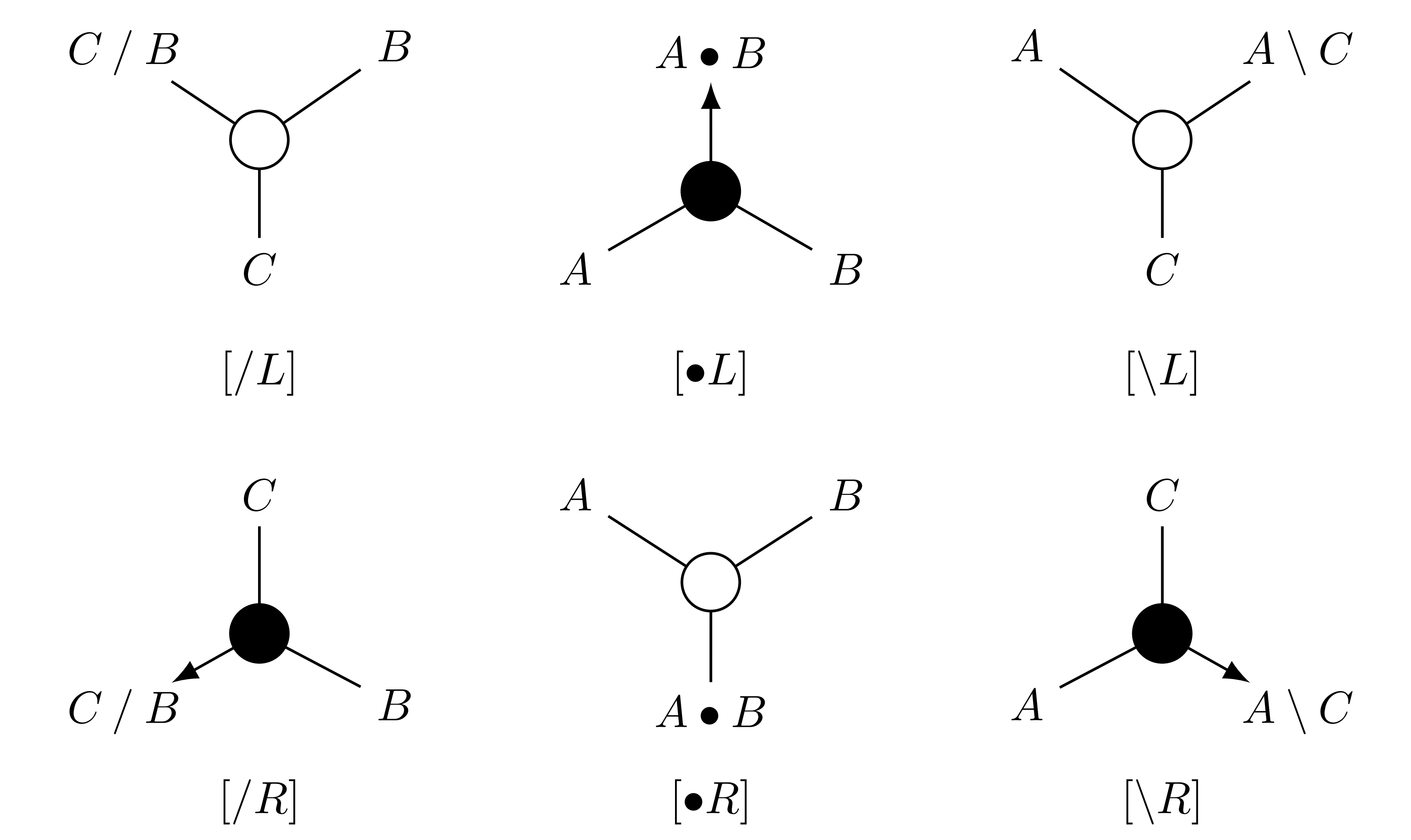
  1. Write down formula decomposition tree

  2. Match atoms (leaves) of opposite polarity

  3. Check correctness of underlying structure using graph rewriting

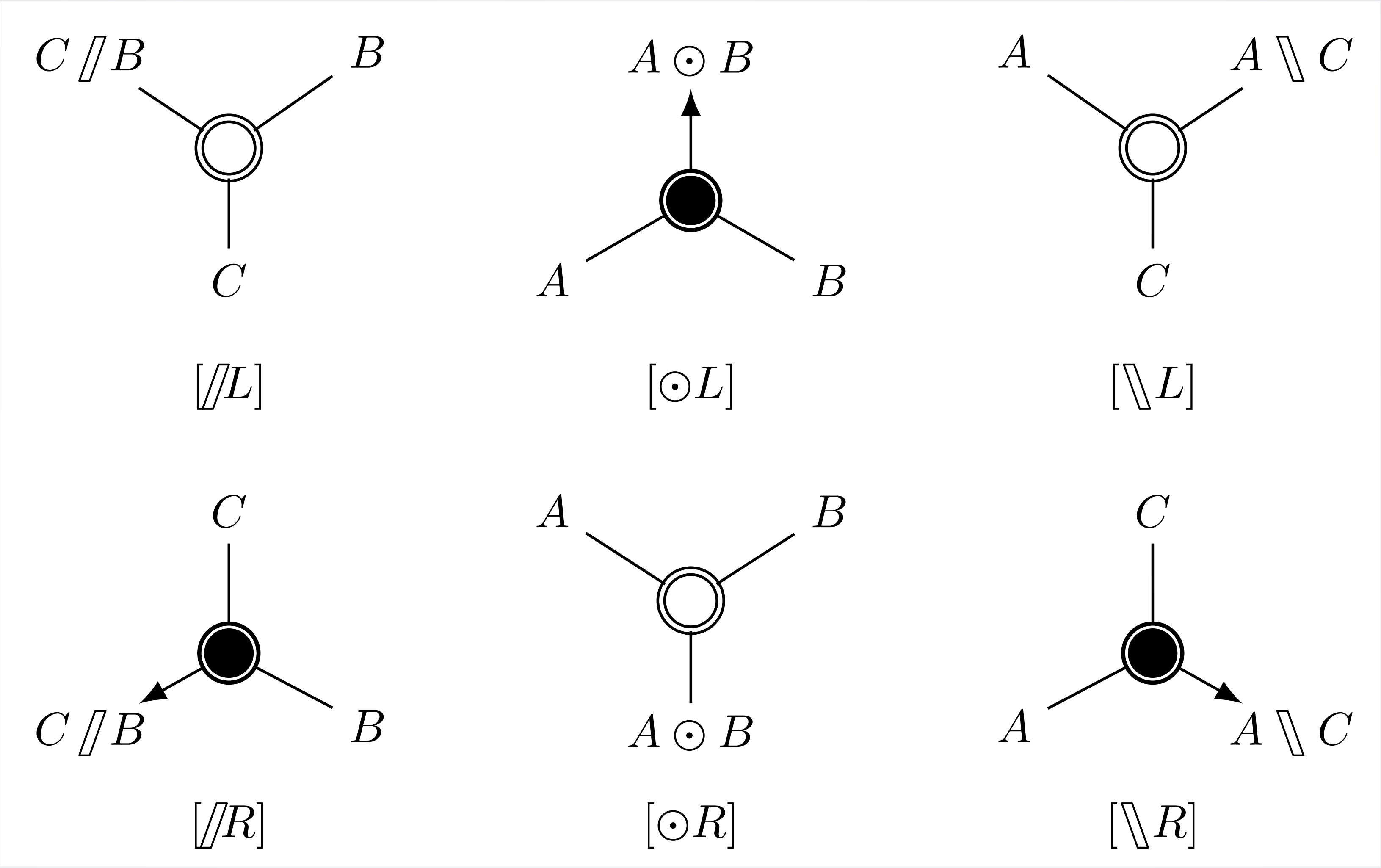
LINKS

NL

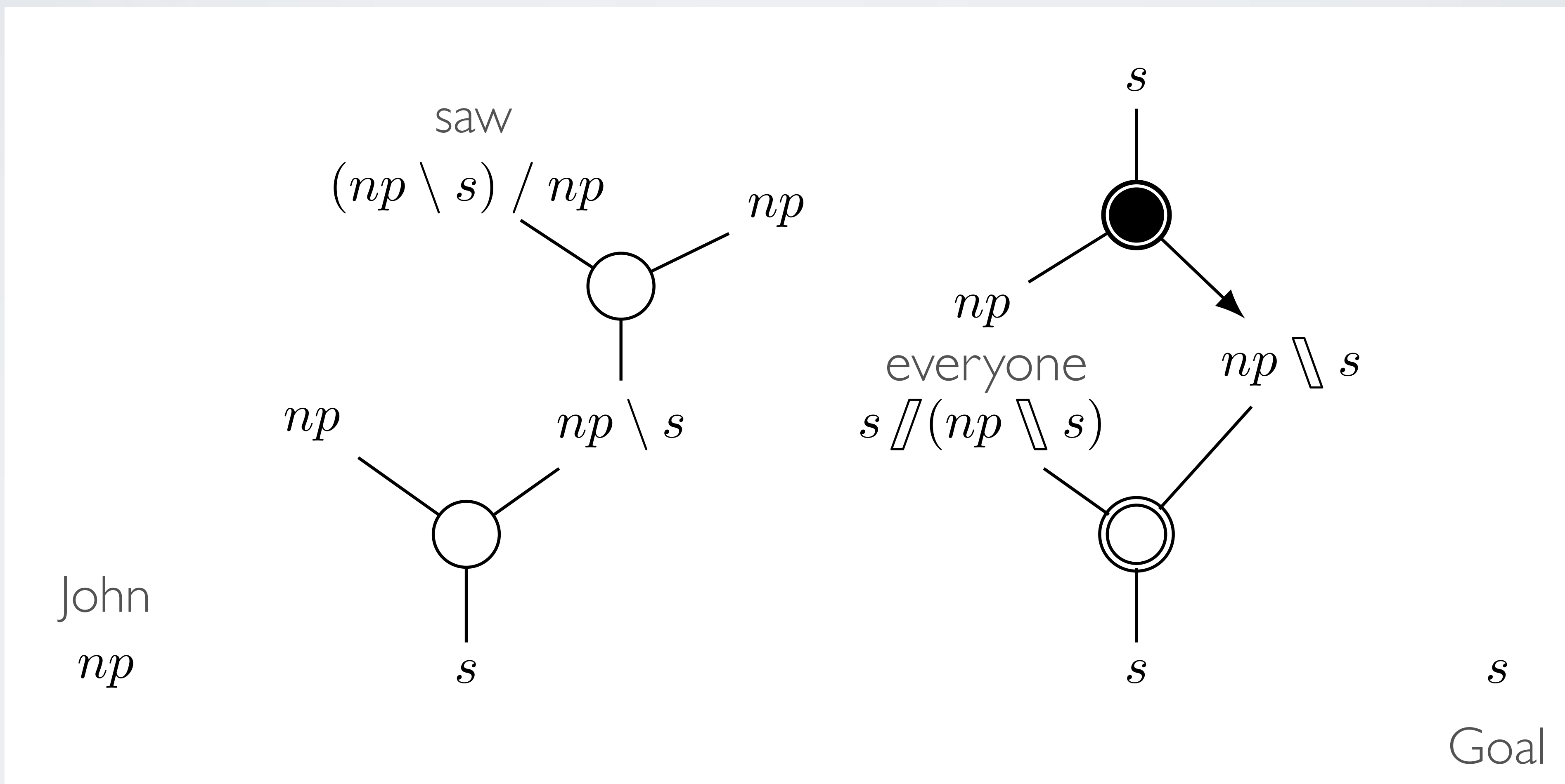

C / B
B
C
$[/L]$
$A \bullet B$
A
B
$[\bullet L]$
A
$A \backslash C$
C
$[\backslash L]$
C
C / B
B
$[/R]$
A
B
$A \bullet B$
$[\bullet R]$
C
A
$A \backslash C$
$[\backslash R]$

LINKS

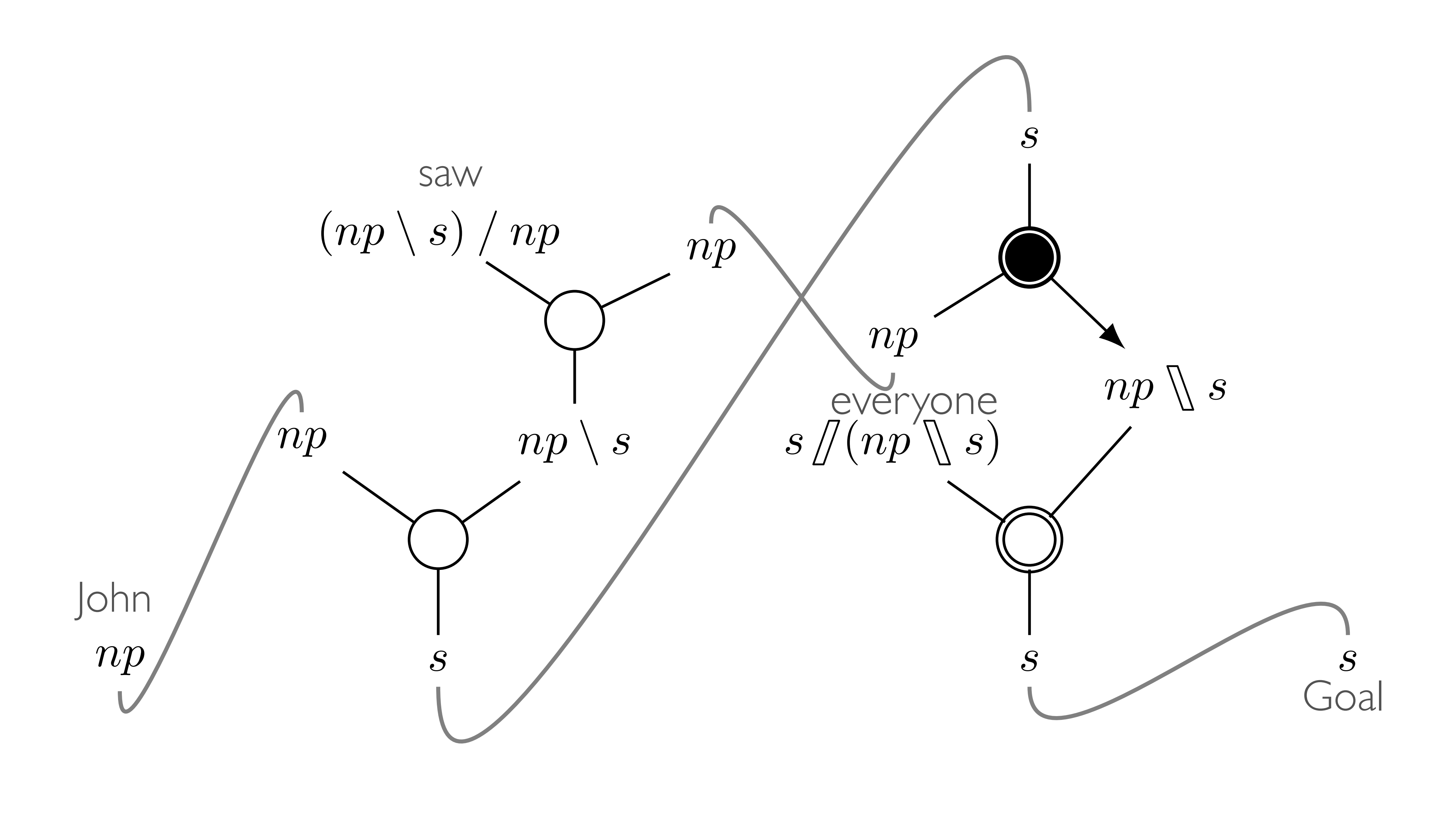
NLλ



EXAMPLE: JOHN SAW EVERYONE



EXAMPLE: JOHN SAW EVERYONE



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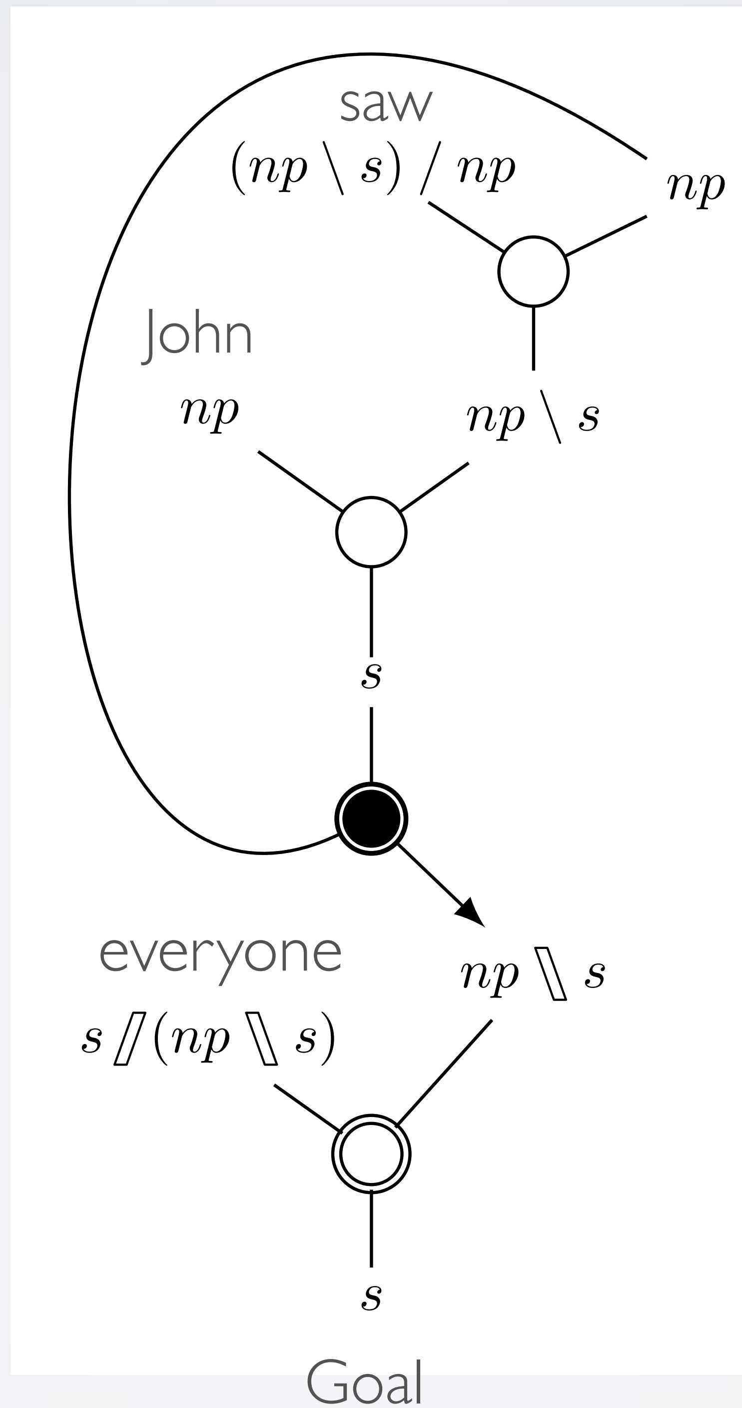
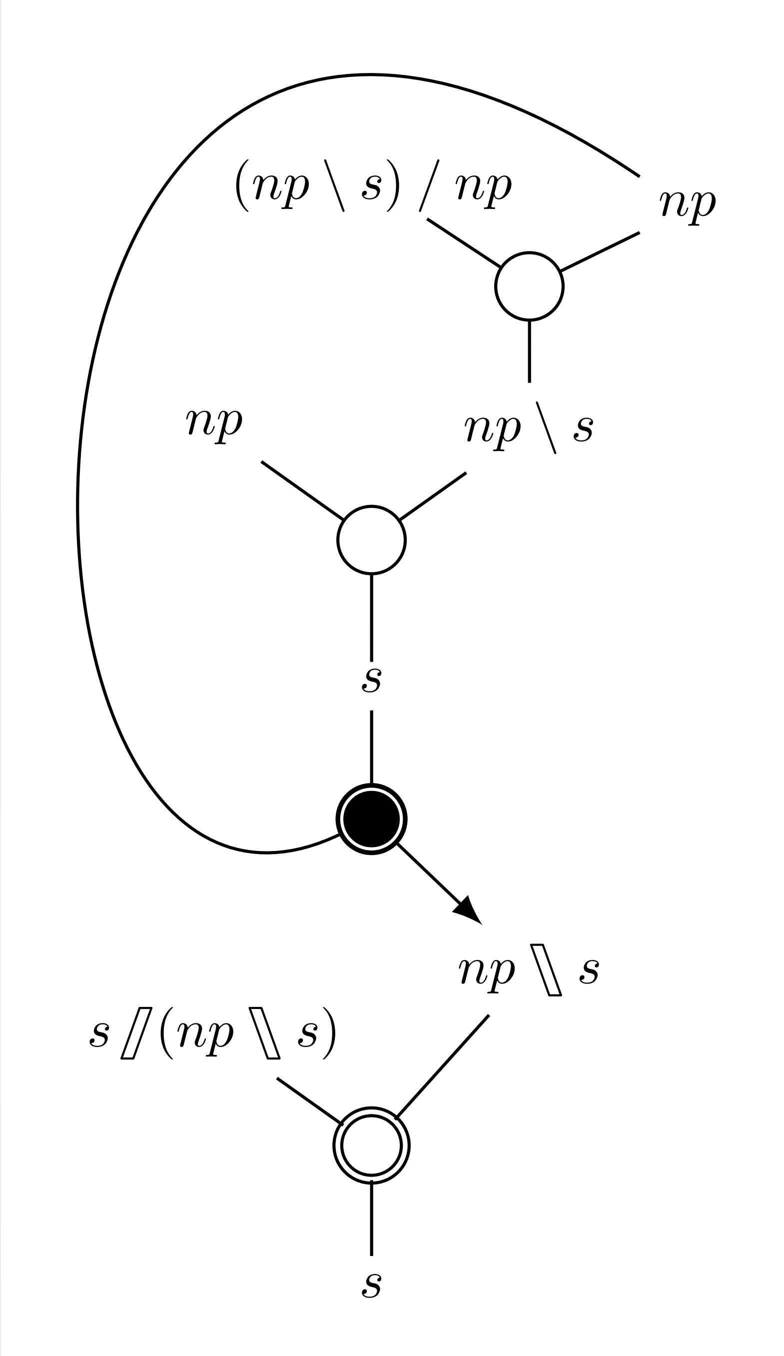
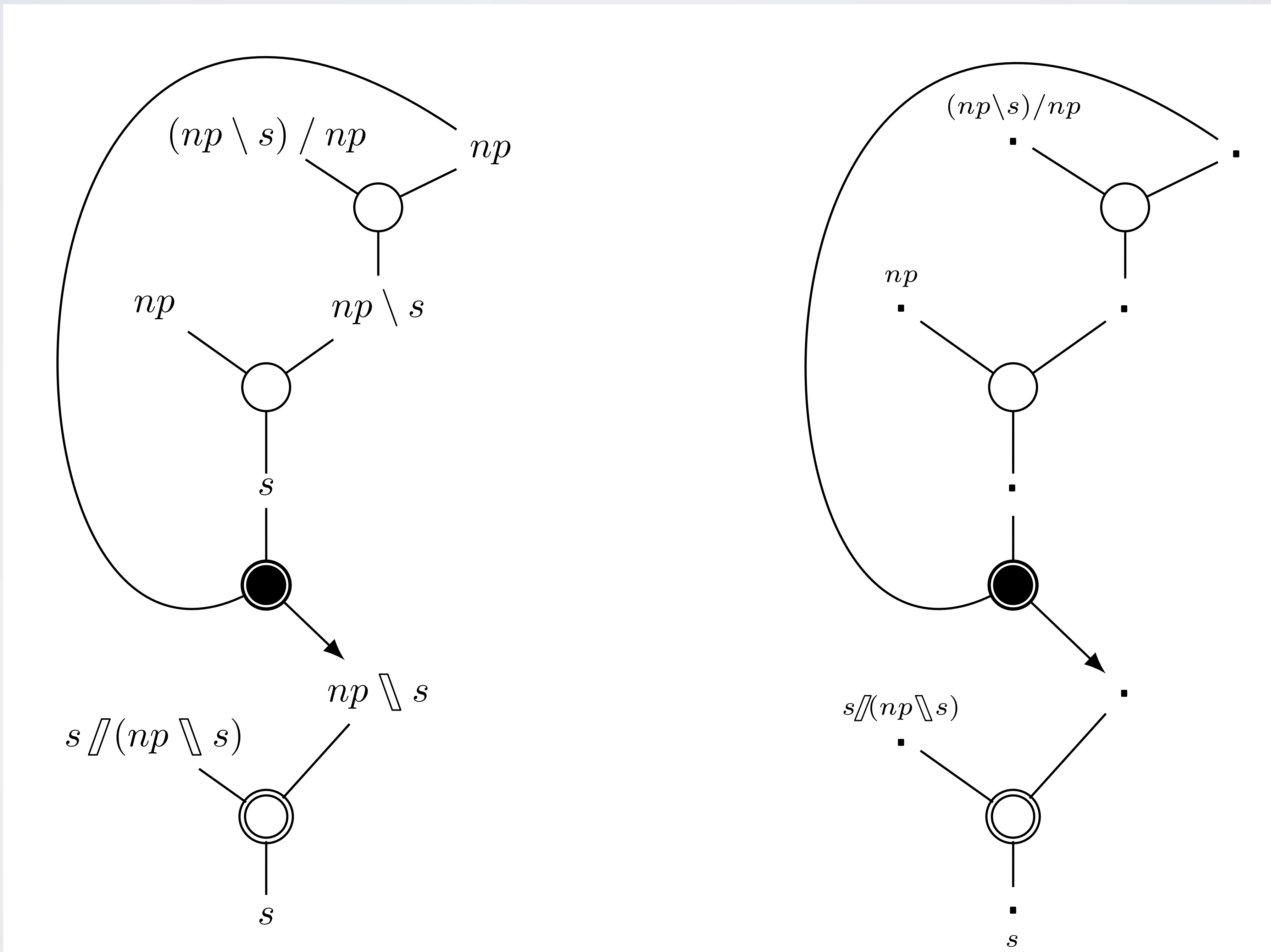


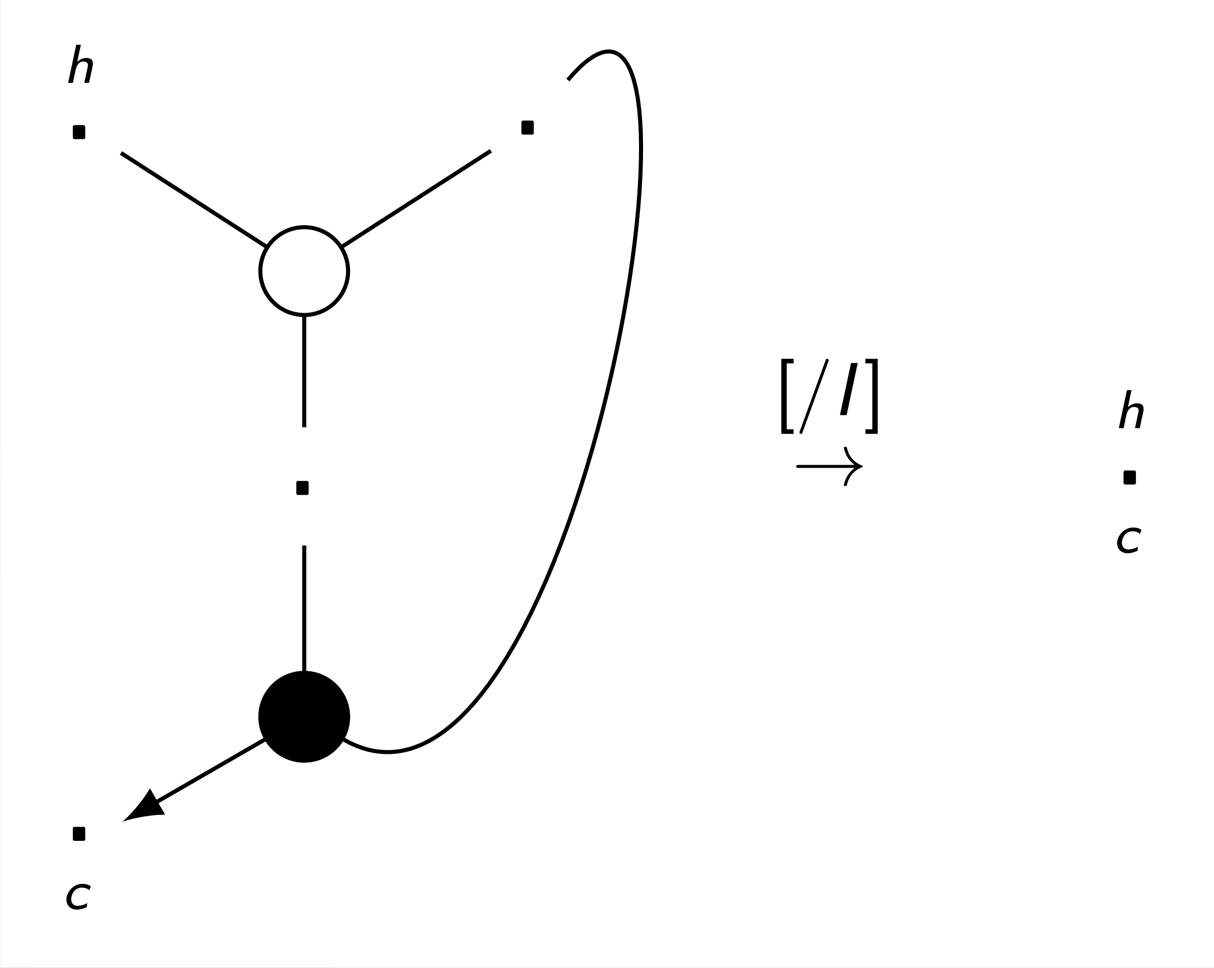
EXAMPLE: JOHN SAW EVERYONE



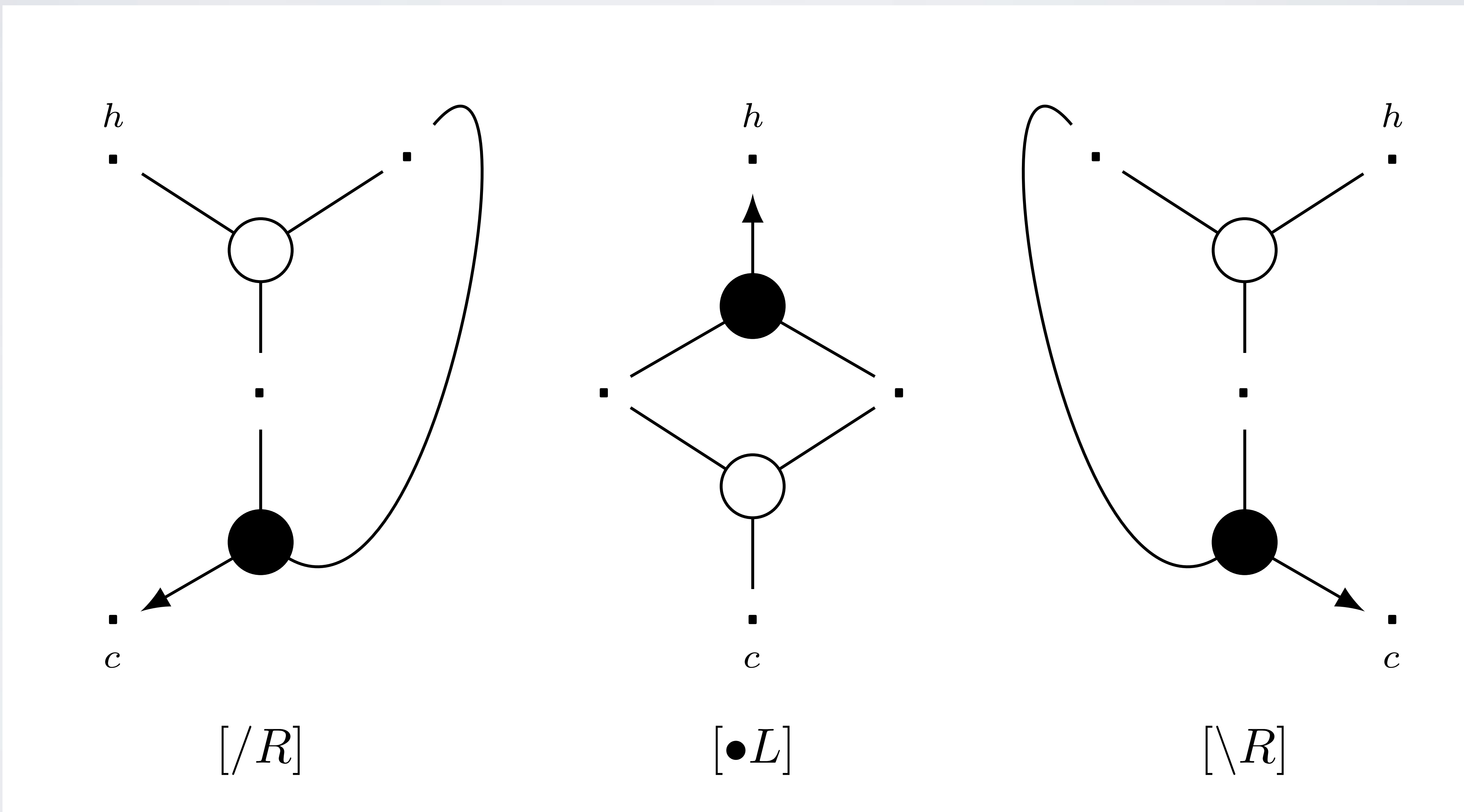
EXAMPLE: JOHN SAW EVERYONE



CONTRACTIONS

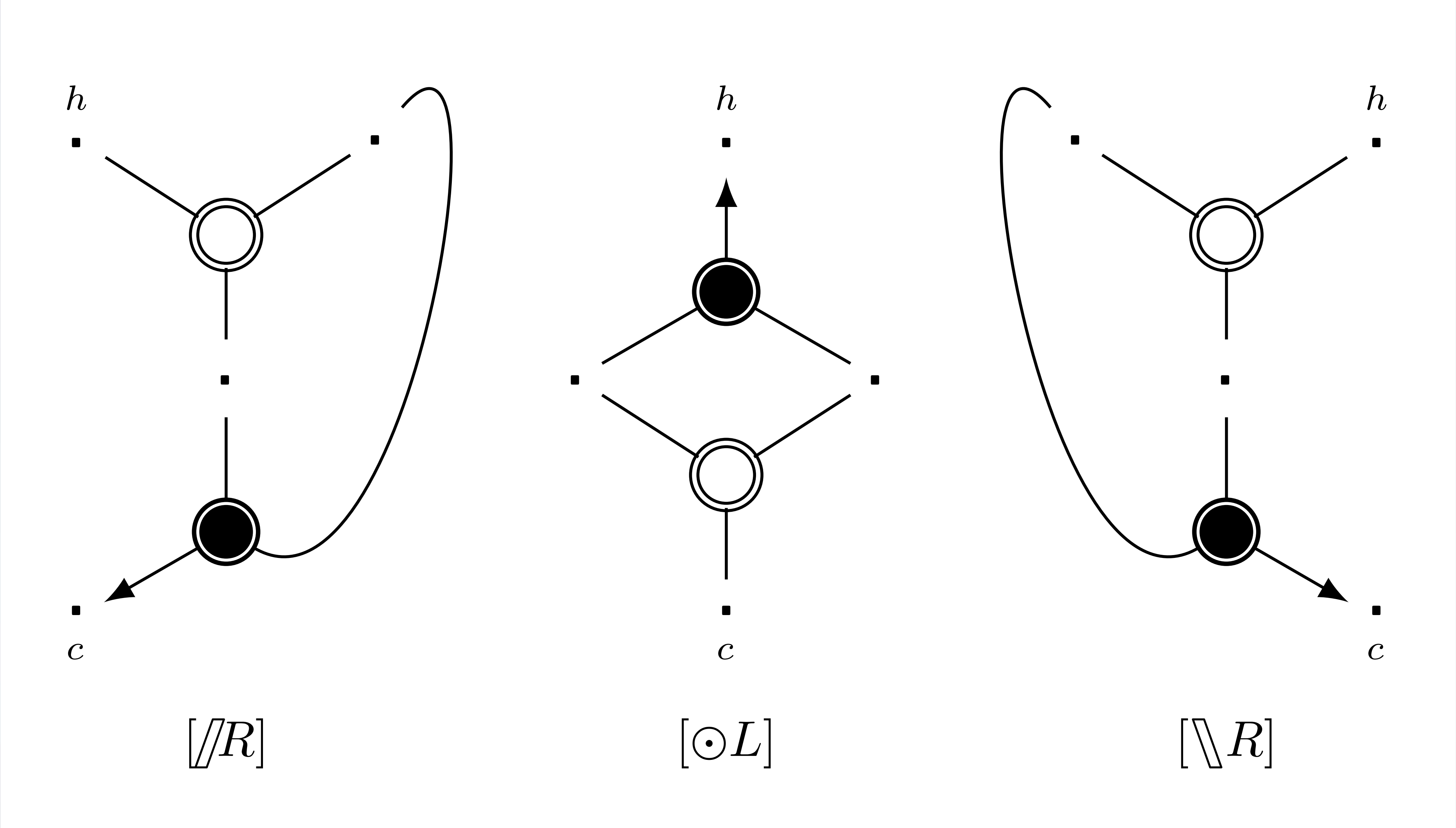


CONTRACTIONS

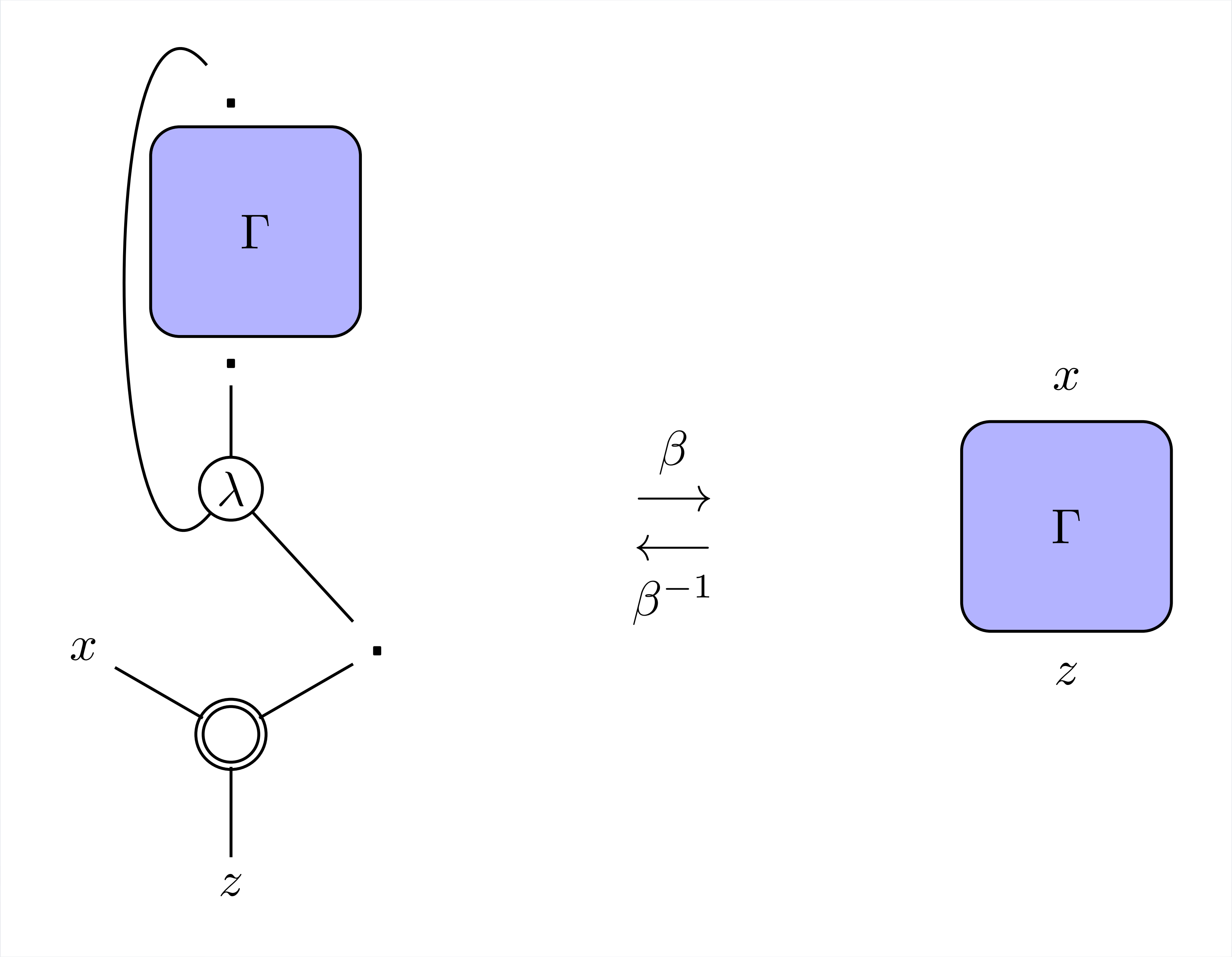


Moot & Puite (2002)

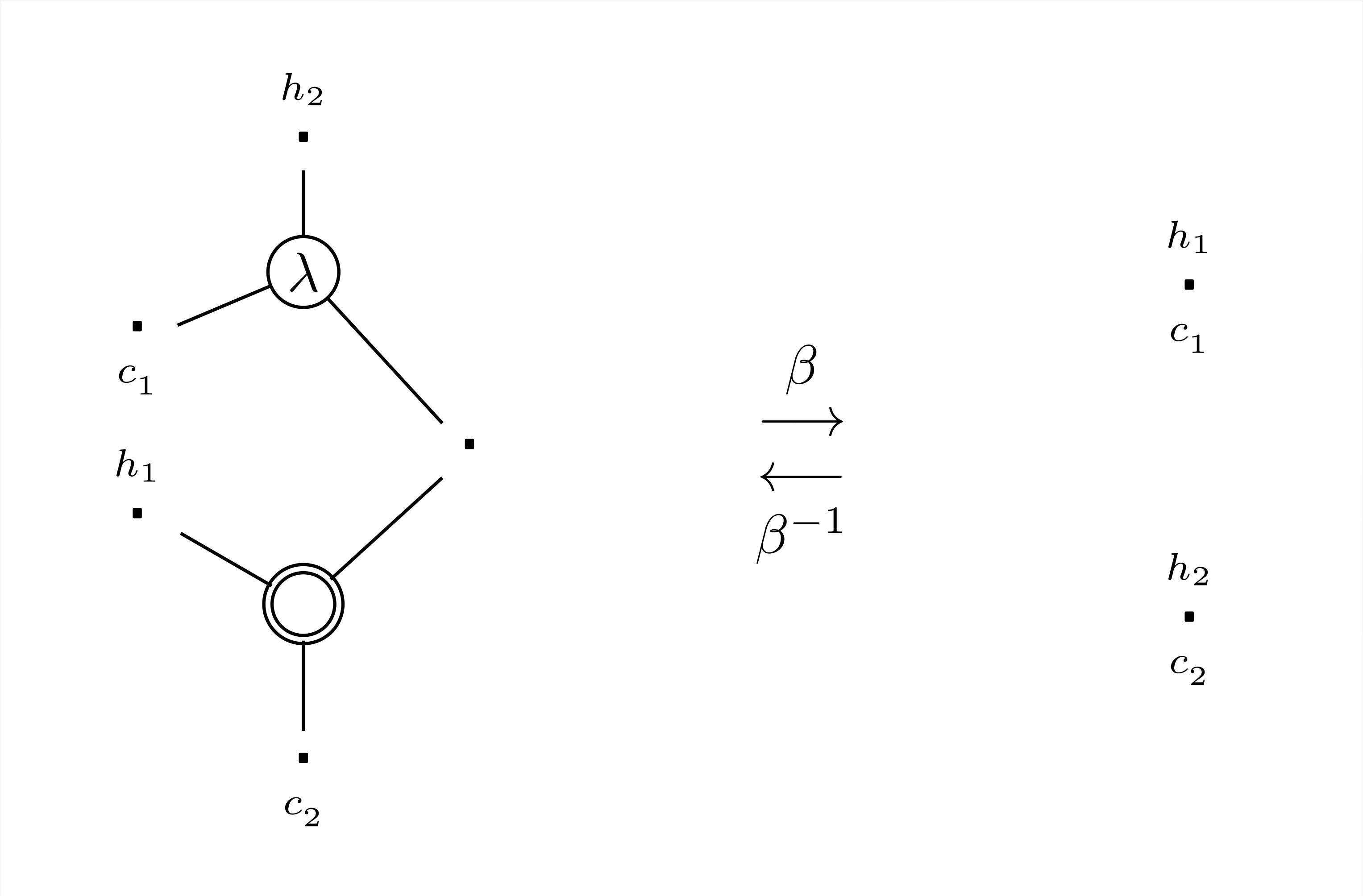
CONTRACTIONS



STRUCTURAL RULES: SUGARED VERSION

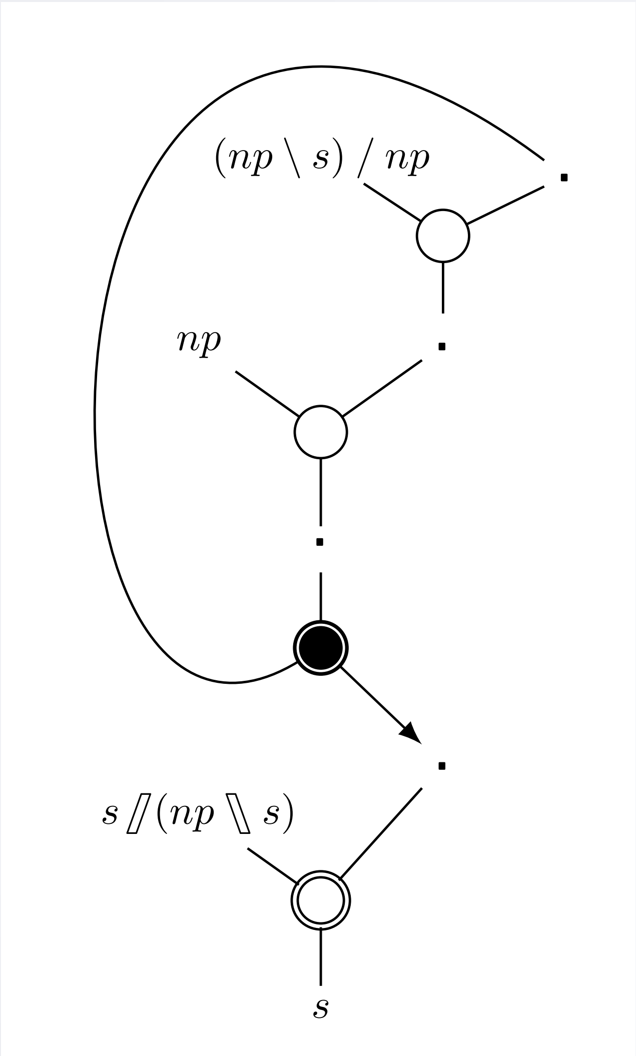


STRUCTURAL RULES

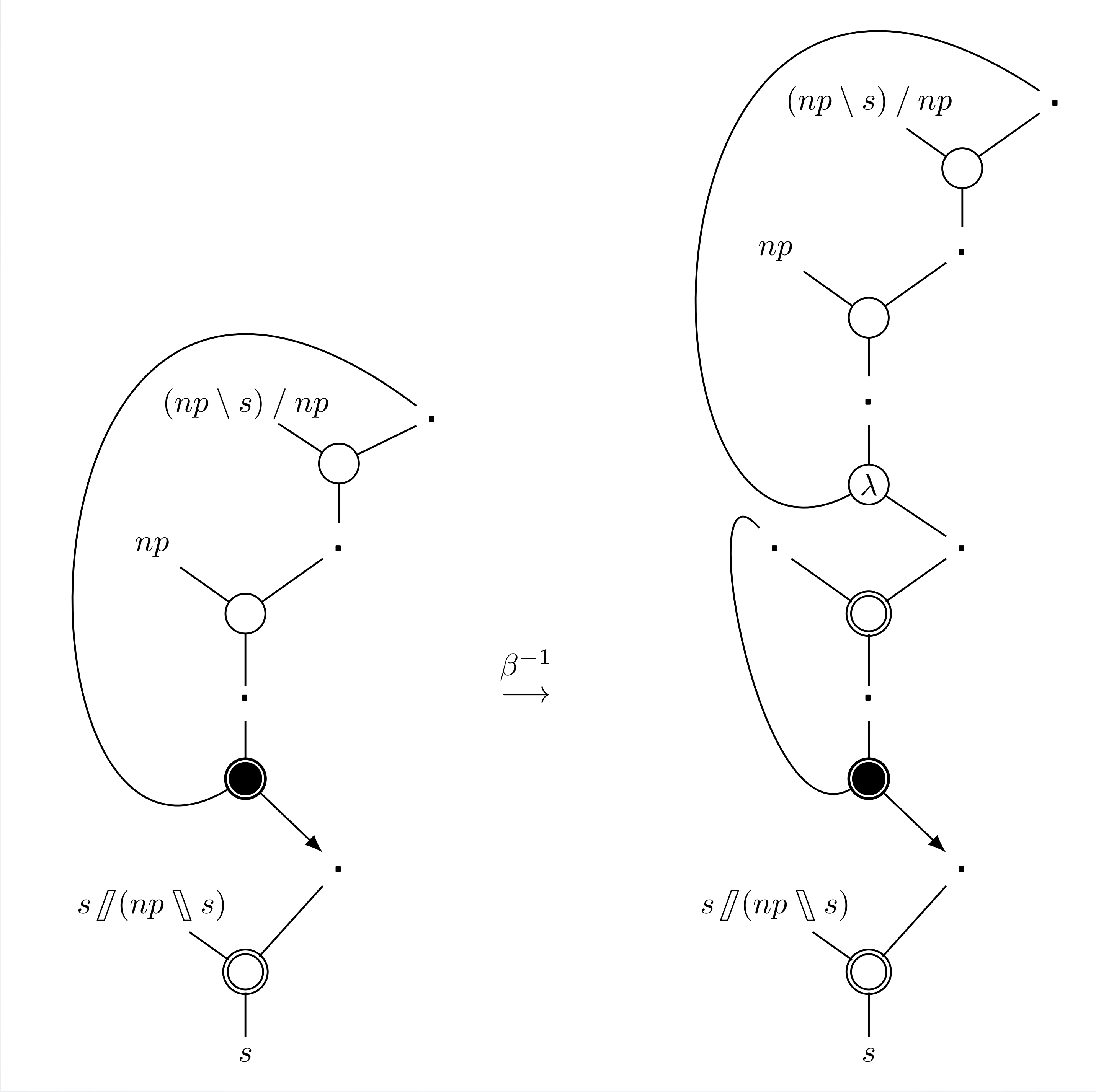


Condition: h_2 must be an ancestor of c_1 by a path which does not pass any asynchronous (par, filled) links

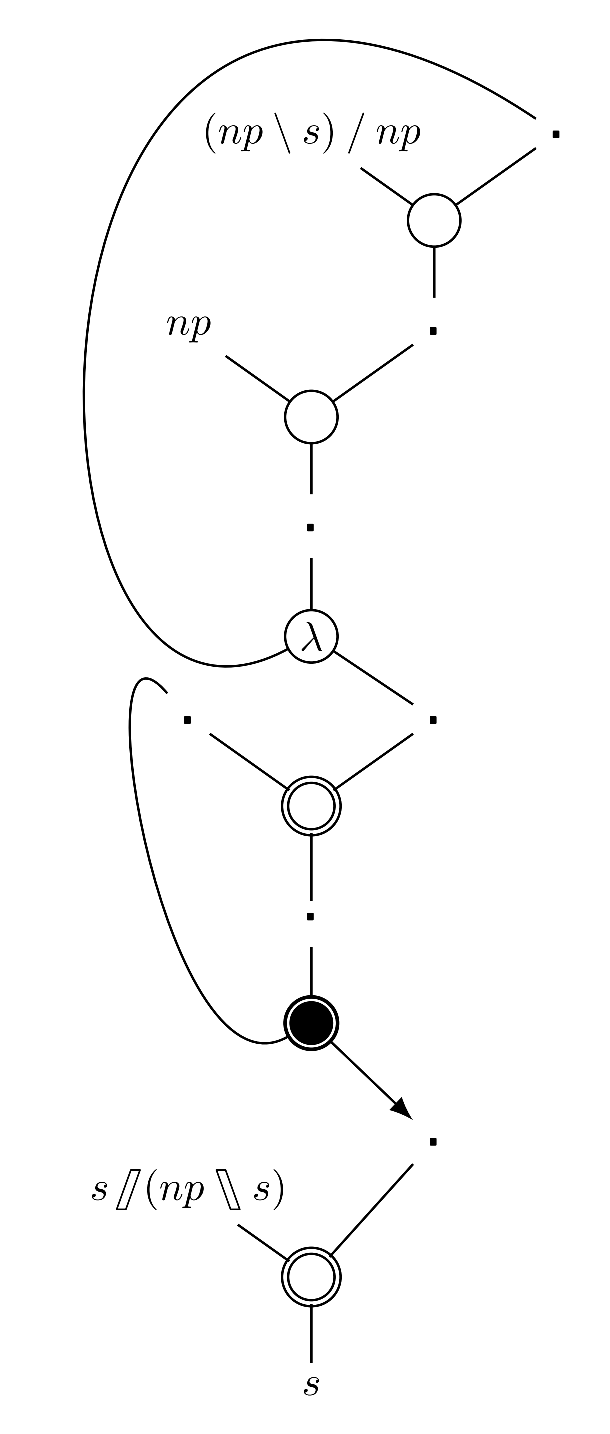
EXAMPLE:
JOHN SAW EVERYONE



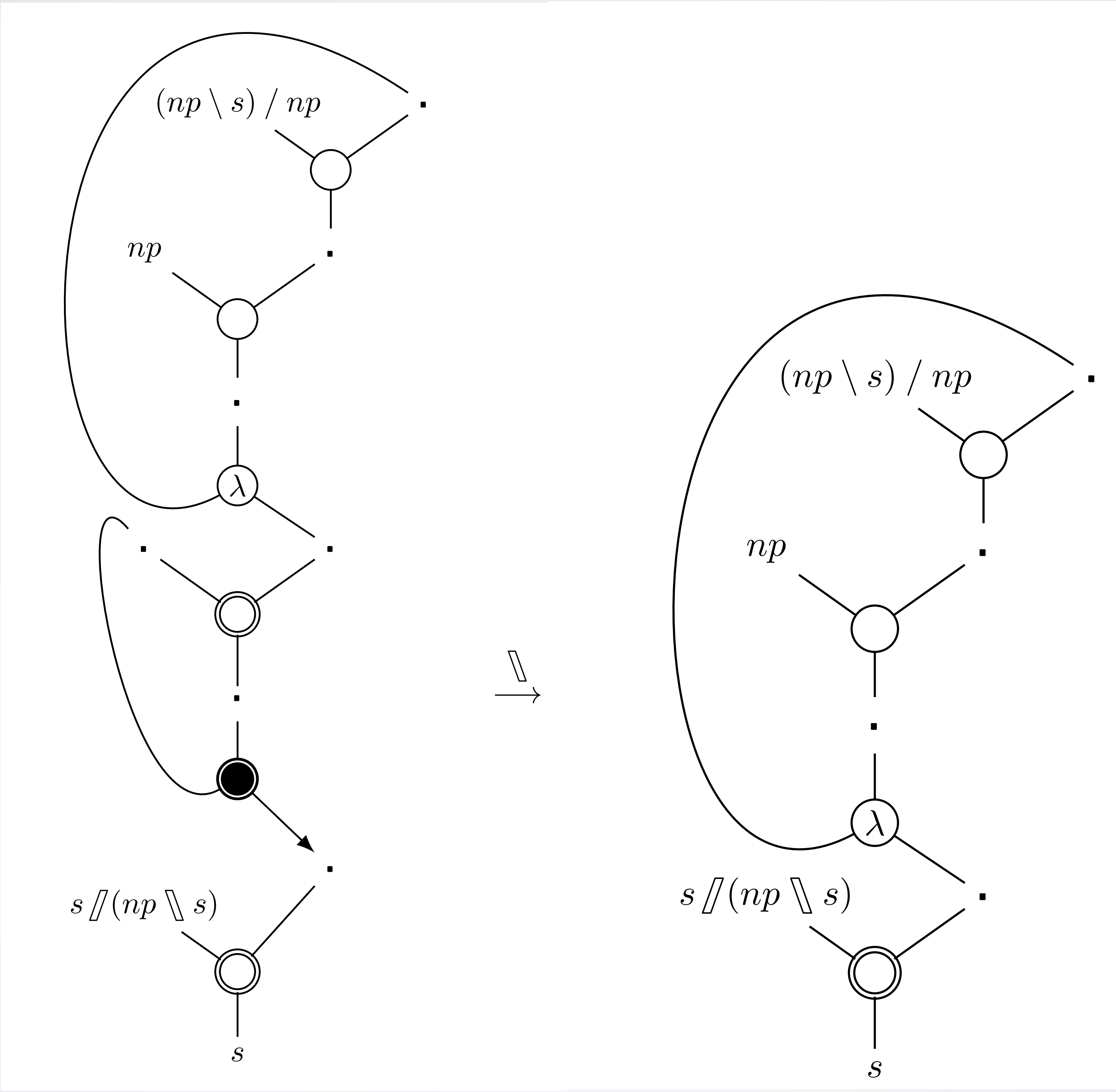
EXAMPLE: JOHN SAW EVERYONE



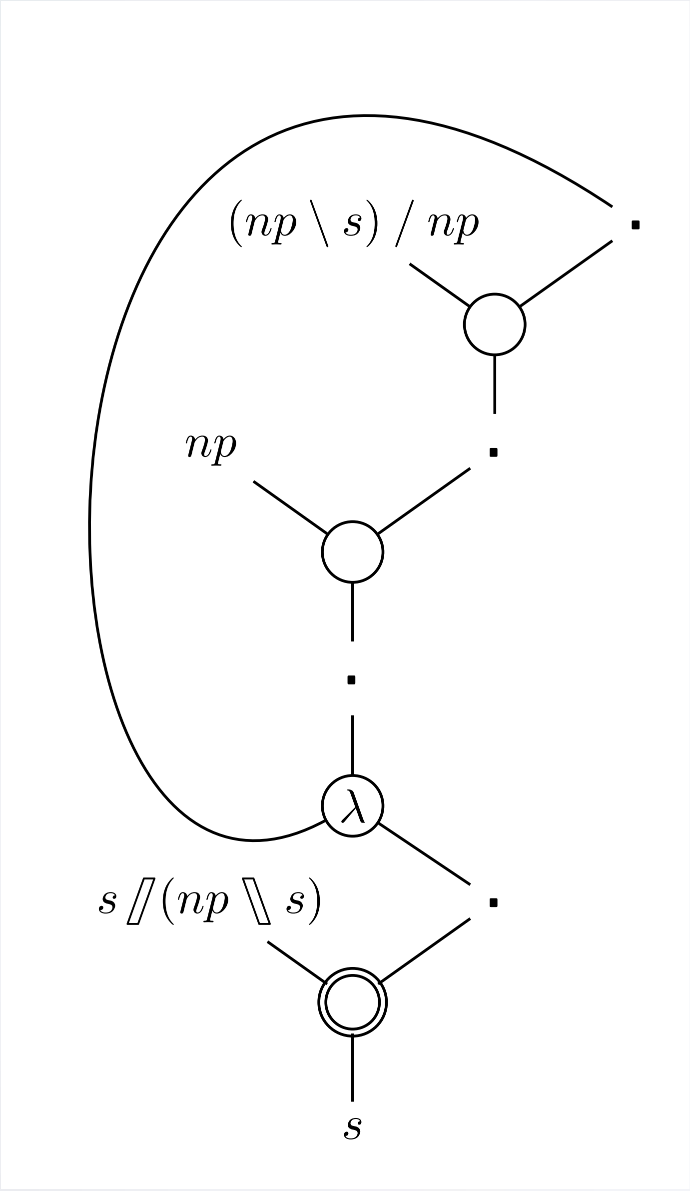
EXAMPLE: JOHN SAW EVERYONE



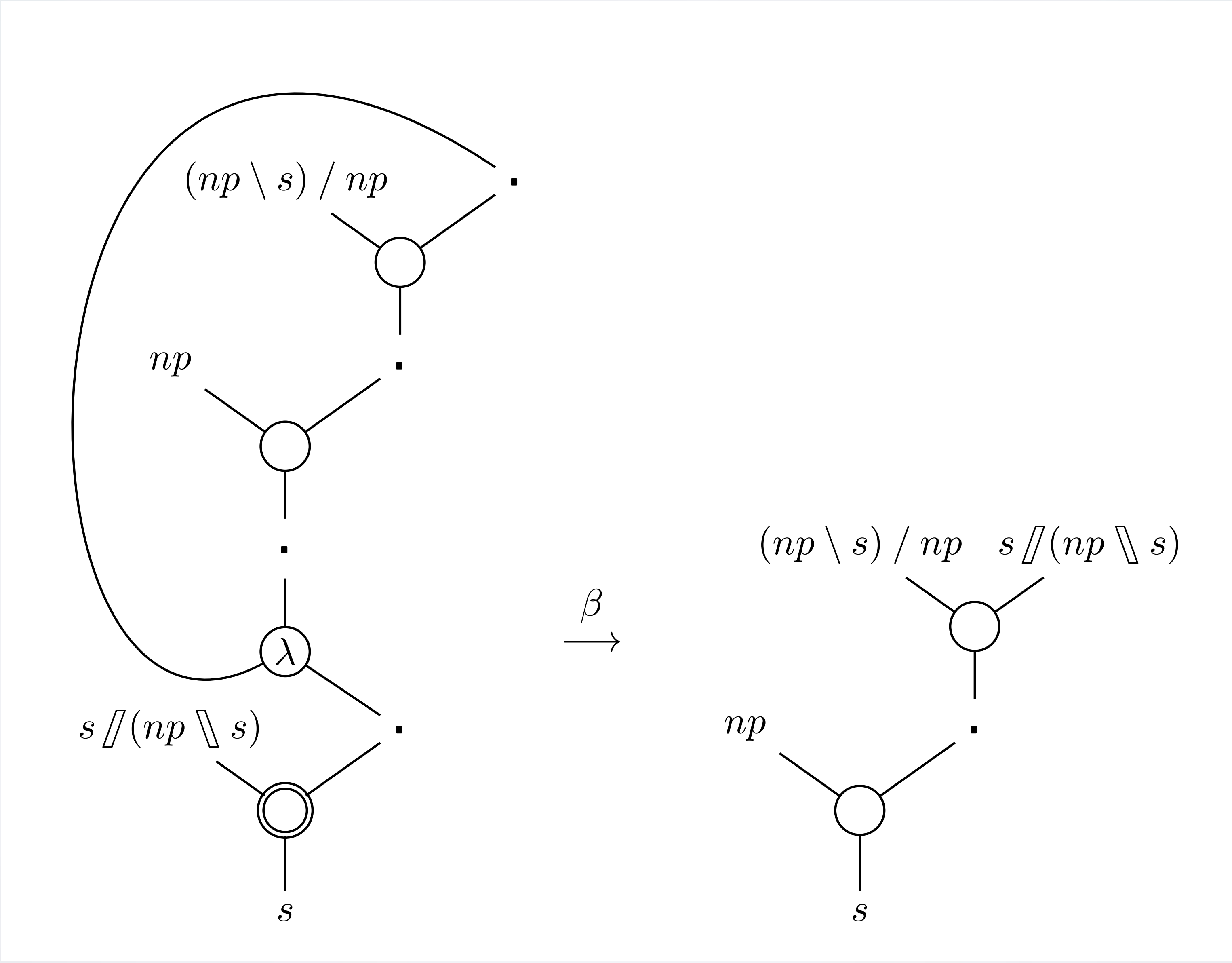
EXAMPLE: JOHN SAW EVERYONE



EXAMPLE: JOHN SAW EVERYONE

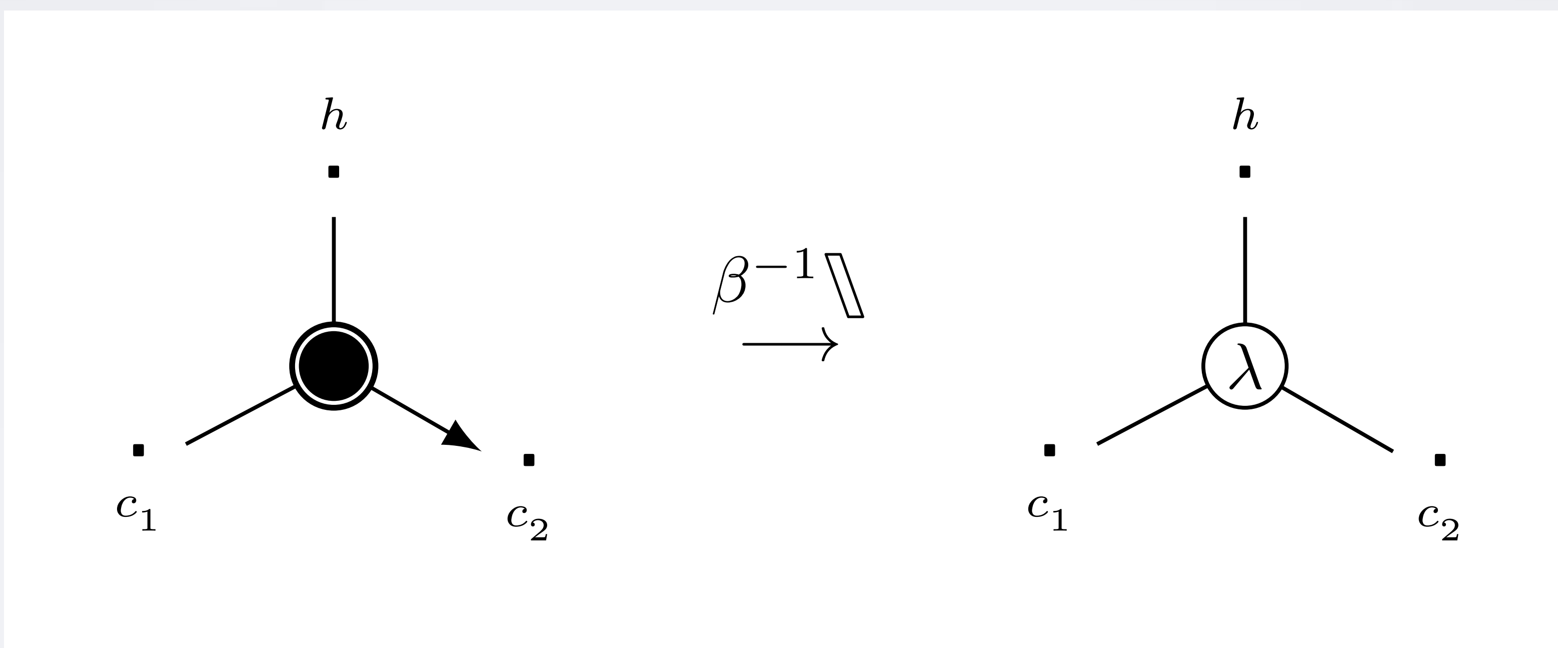


EXAMPLE:
JOHN SAW EVERYONE



KEY PROPERTY

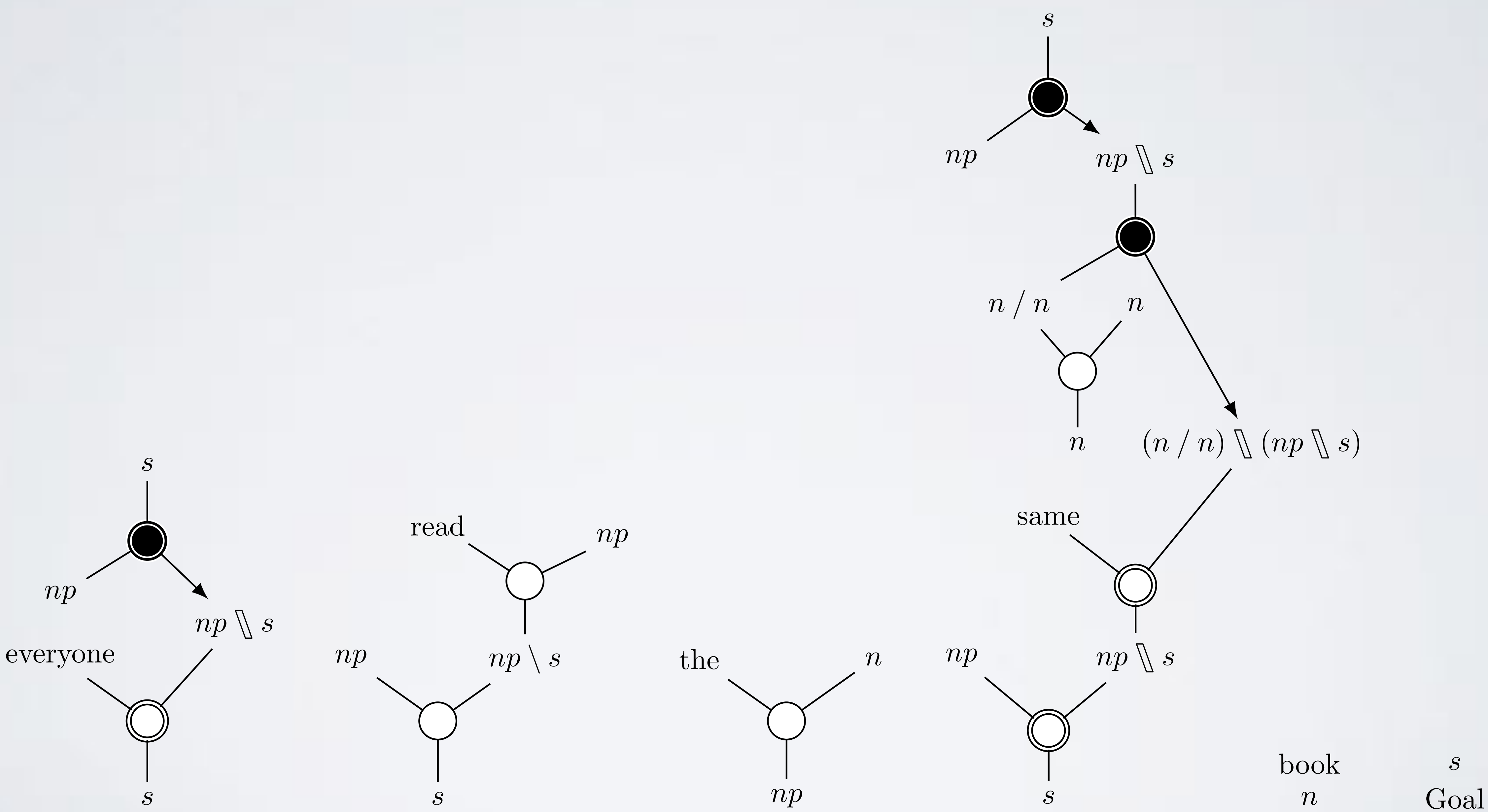
We can, without loss of generality, replace the beta expansion rule by the following rule (a proof net refection of the same principle of Barker 2019)



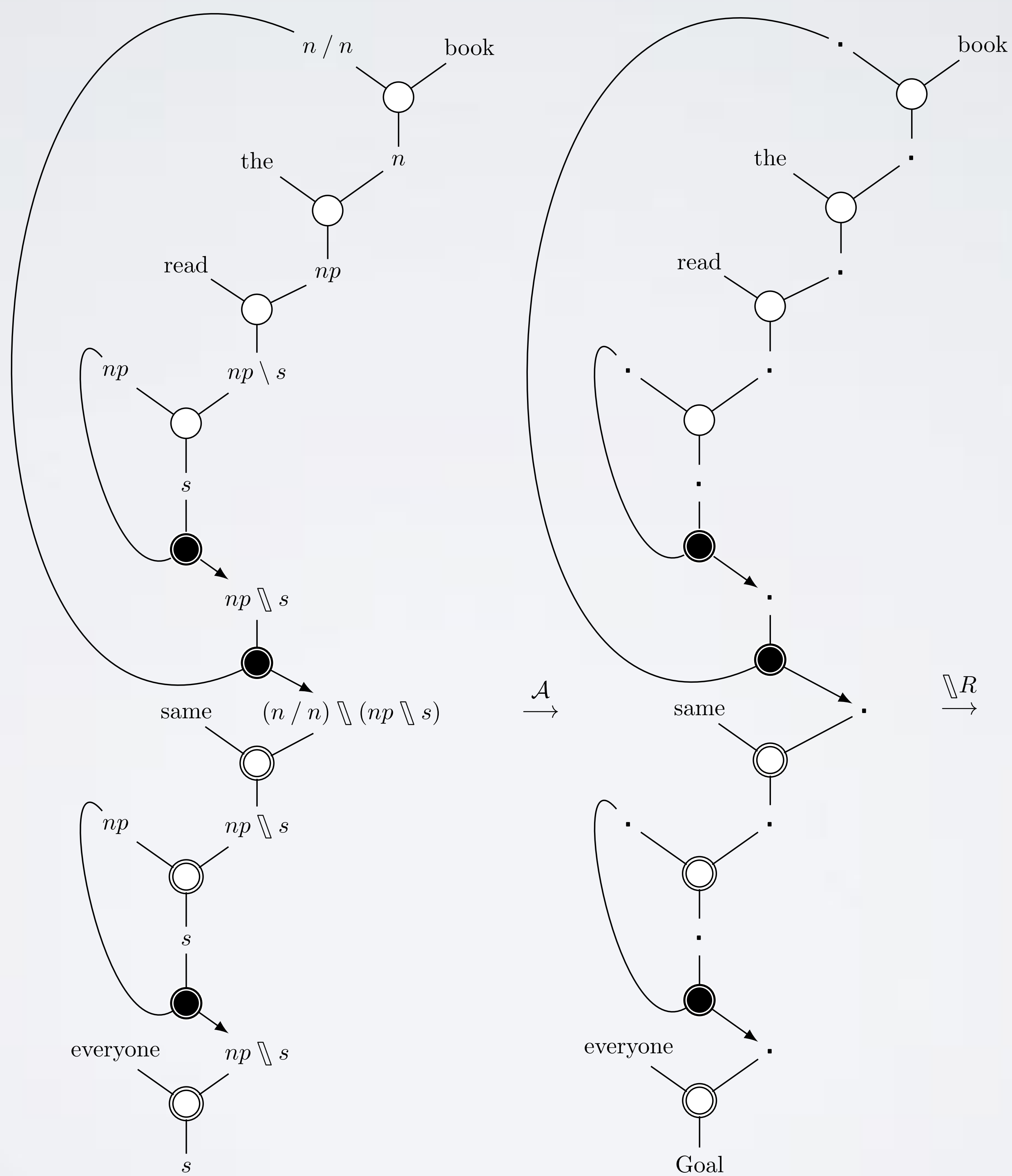
Condition: h must be an ancestor of c_1 by a path which does not pass any asynchronous (par) links

skip example

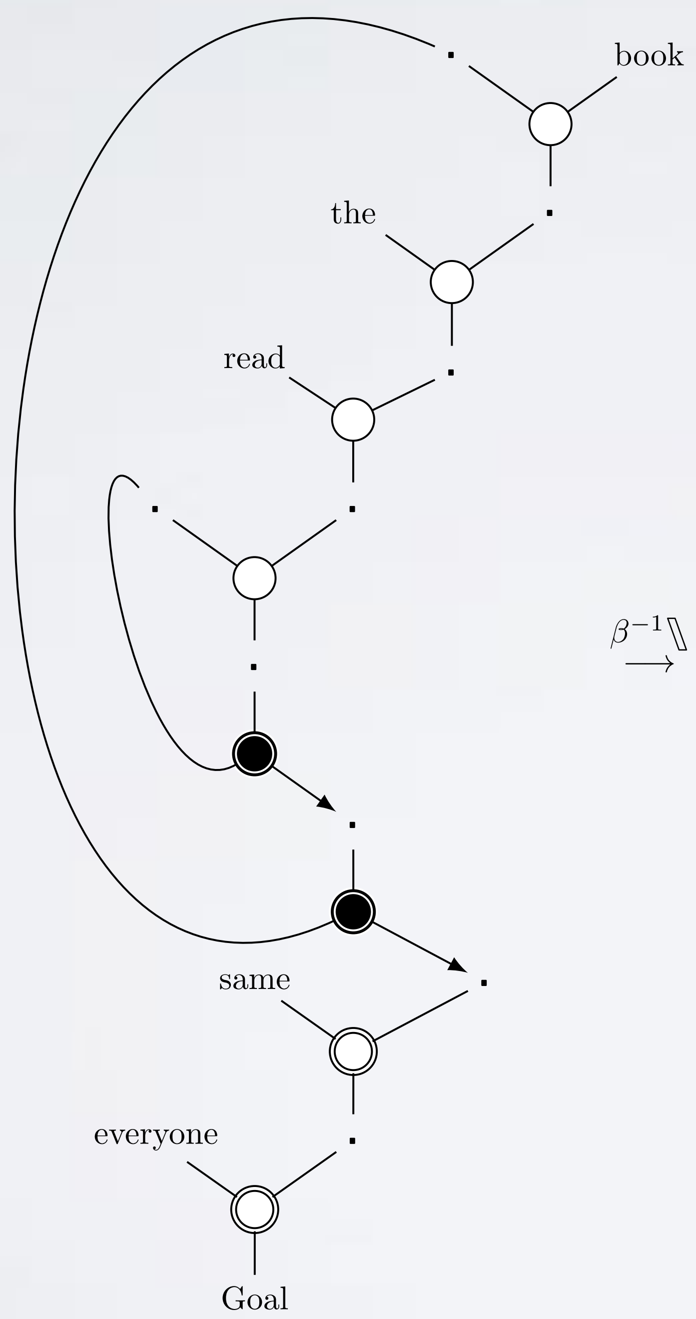
EVERYONE READ THE SAME BOOK

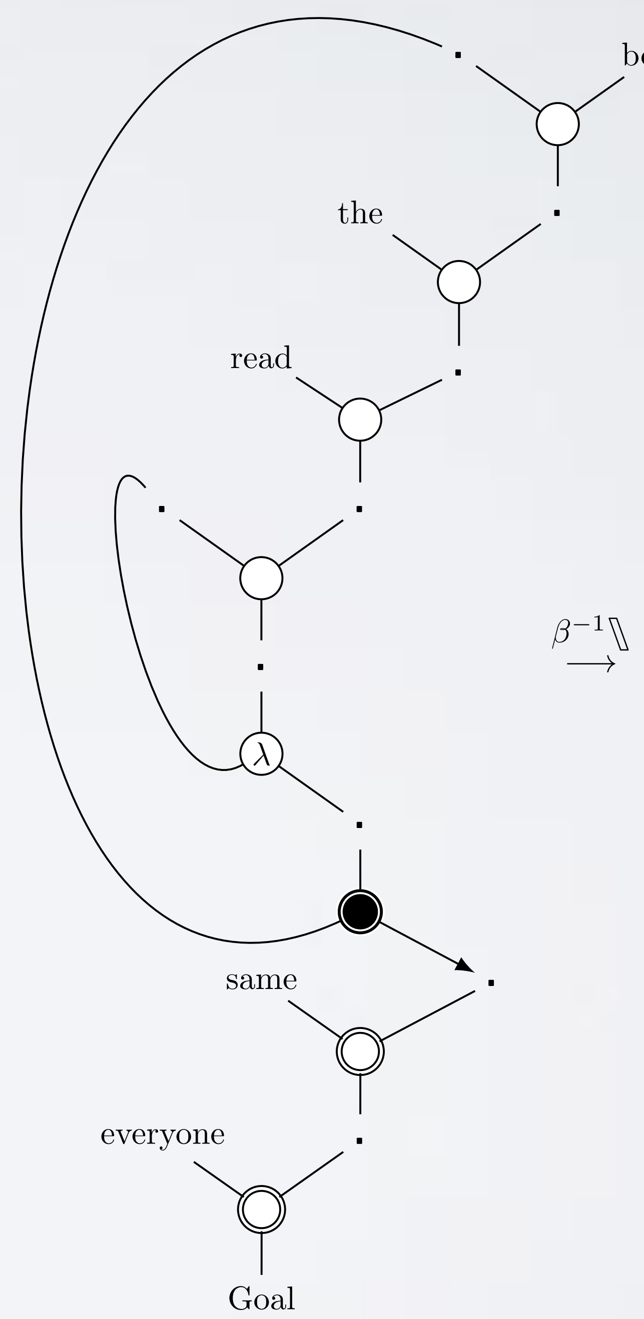


EVERYONE READ THE SAME BOOK

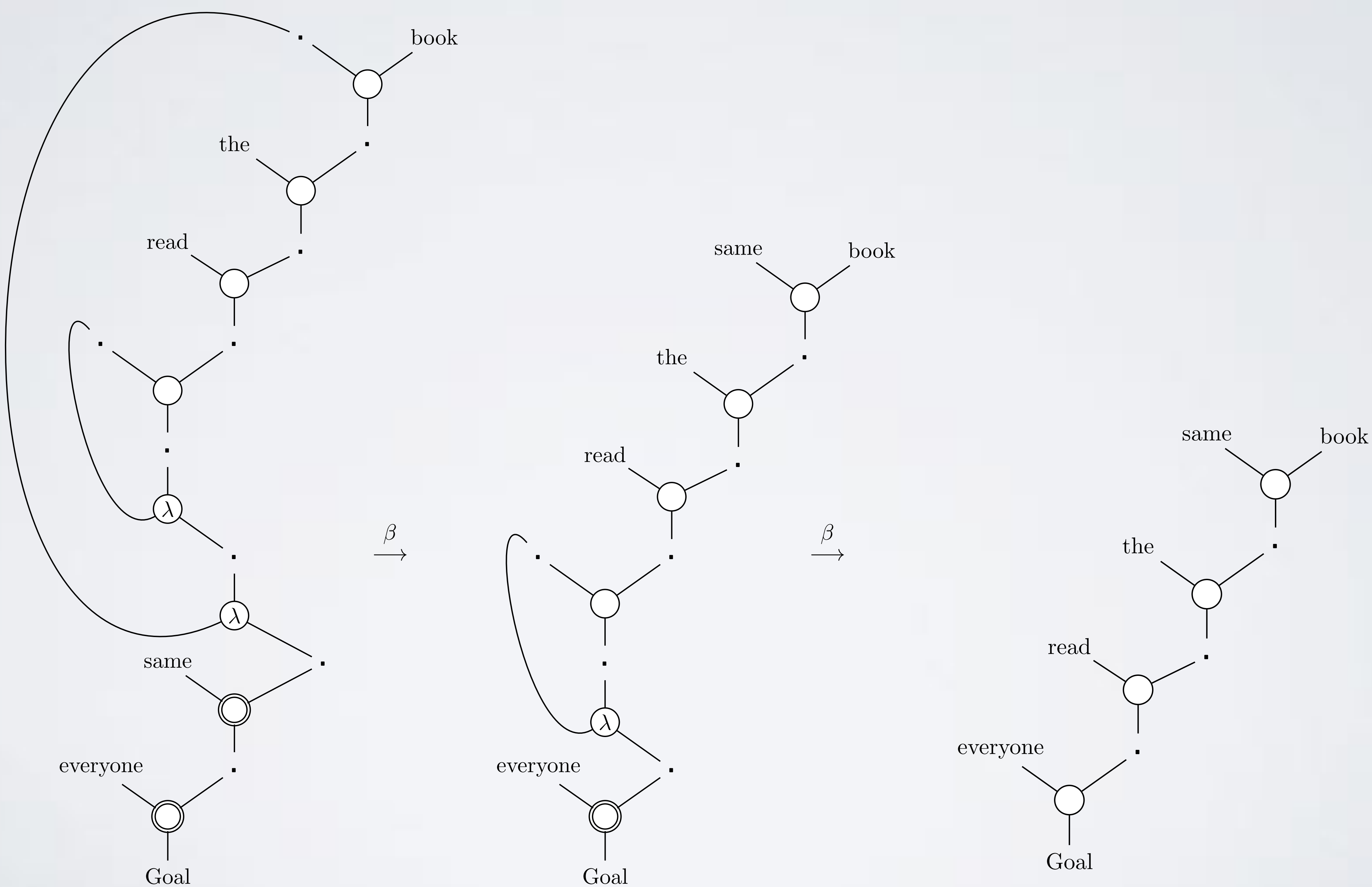


EVERYONE READ THE SAME BOOK

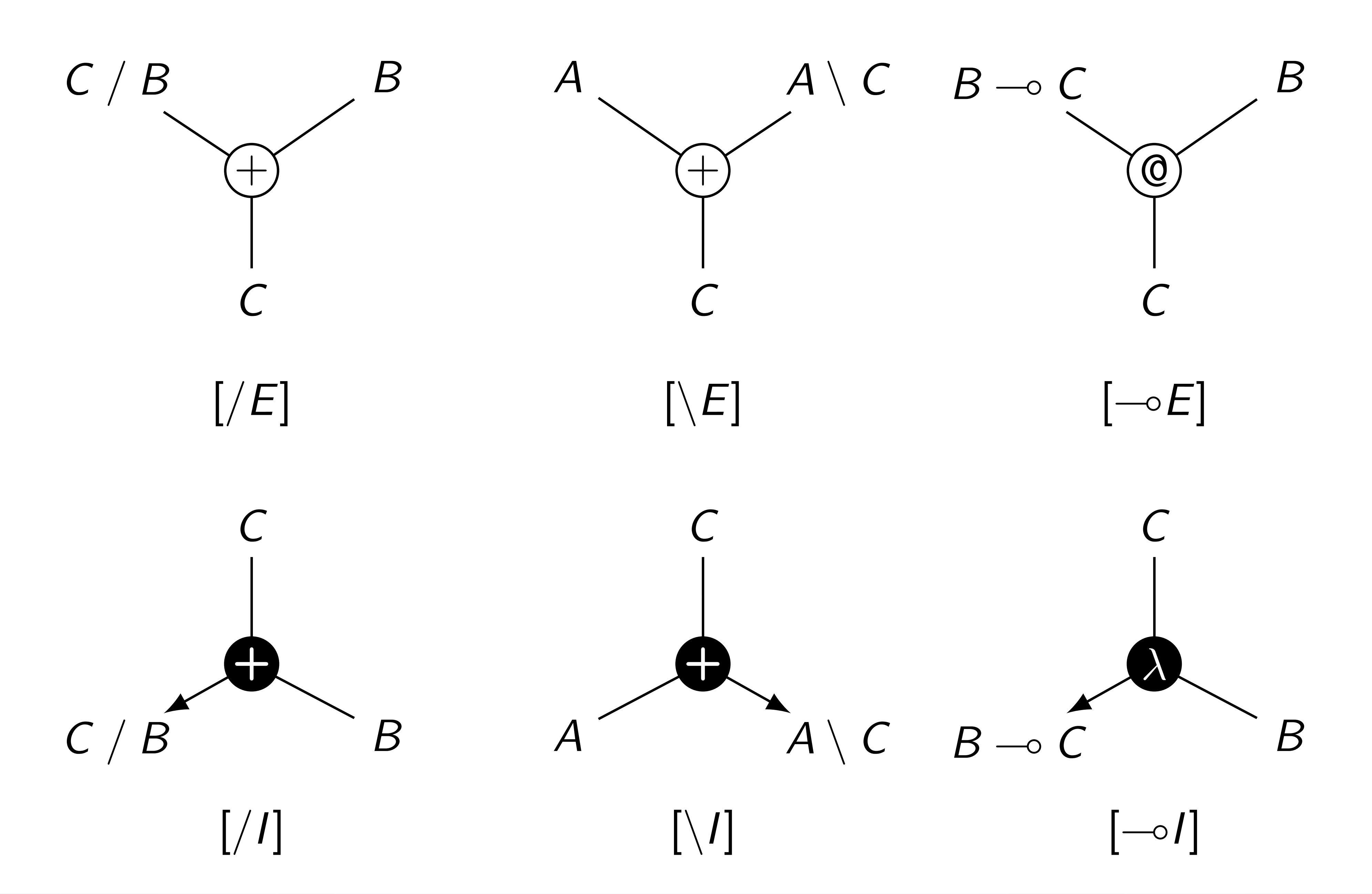




EVERYONE READ THE SAME BOOK

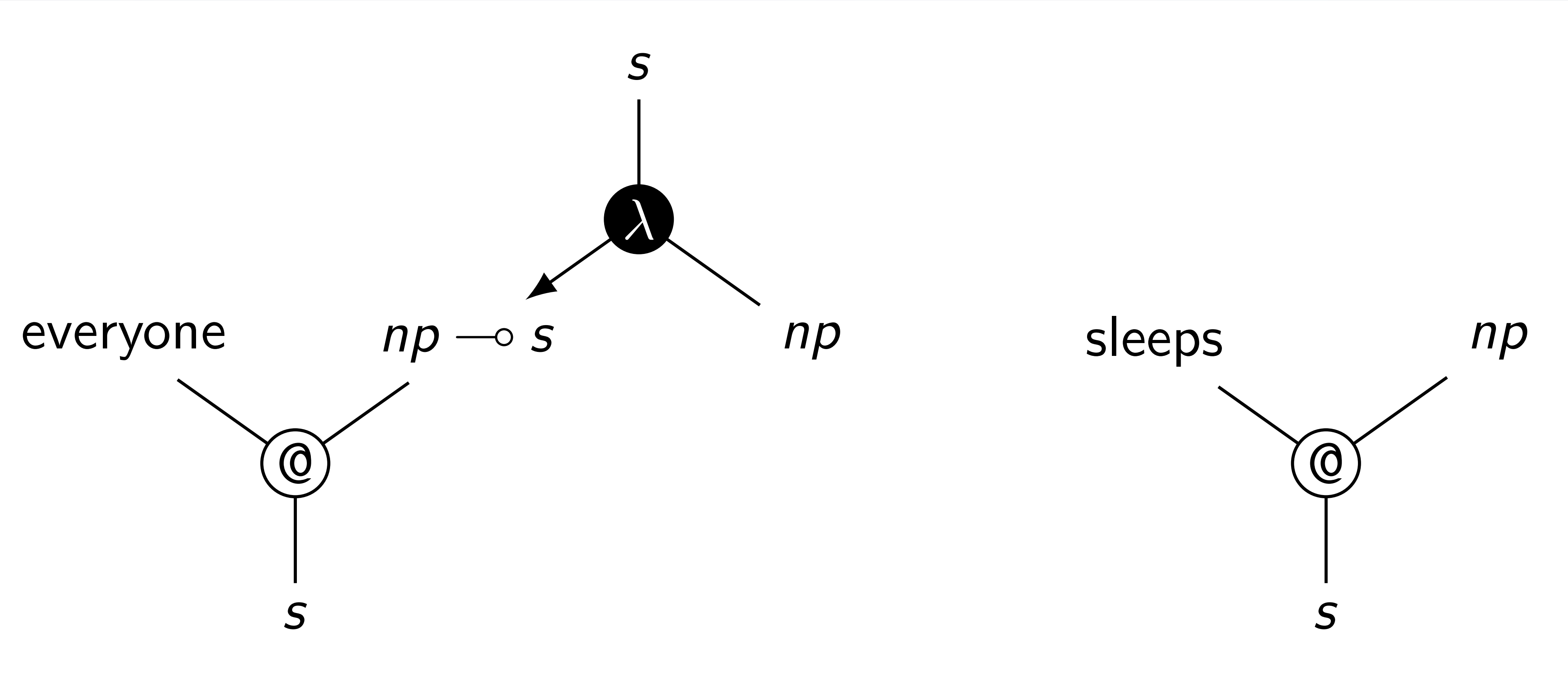


HTLG: LINKS



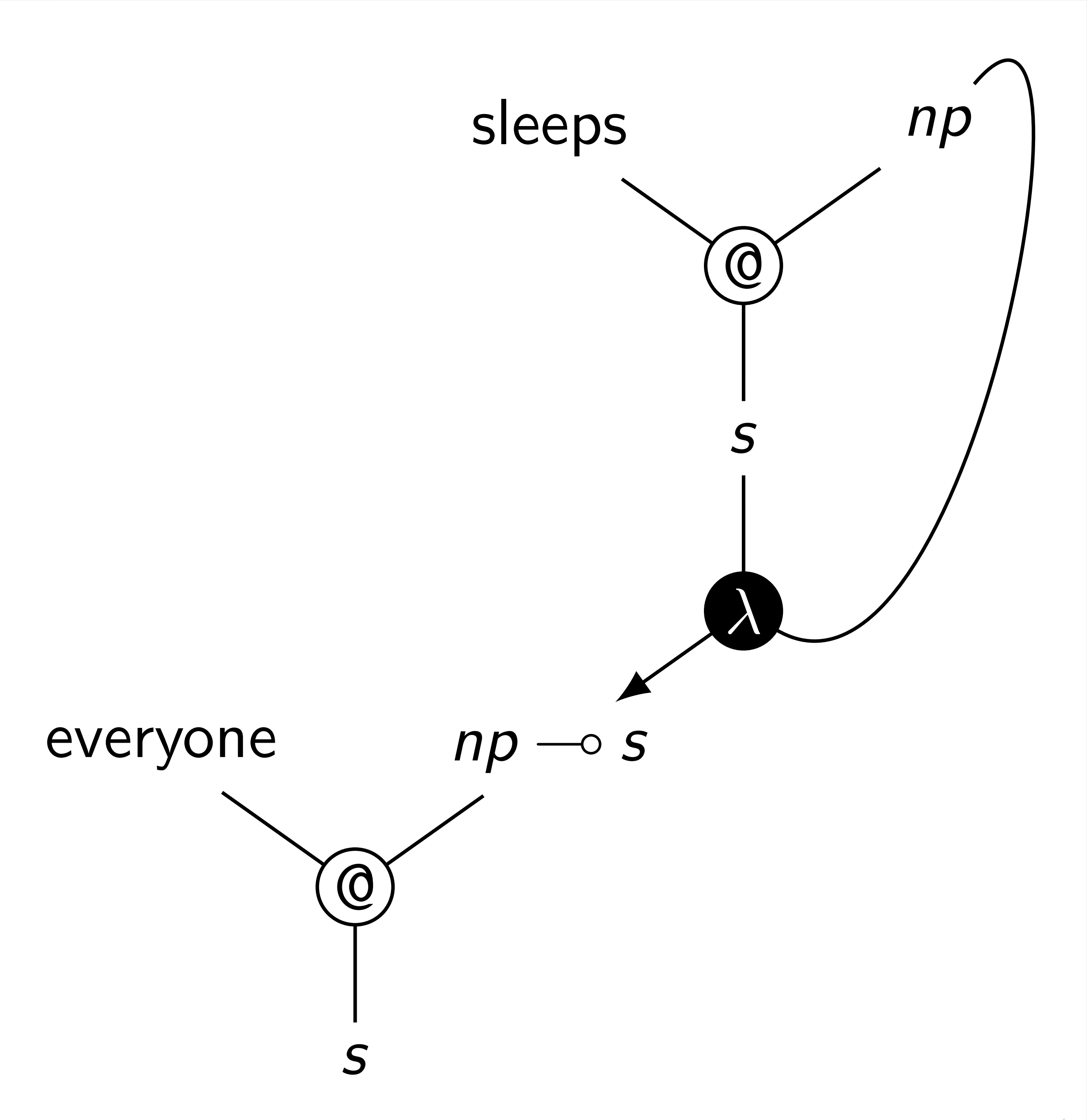
Moot & Stevens-Guile (2019, to appear)

HTLG: EVERYONE SLEEPS

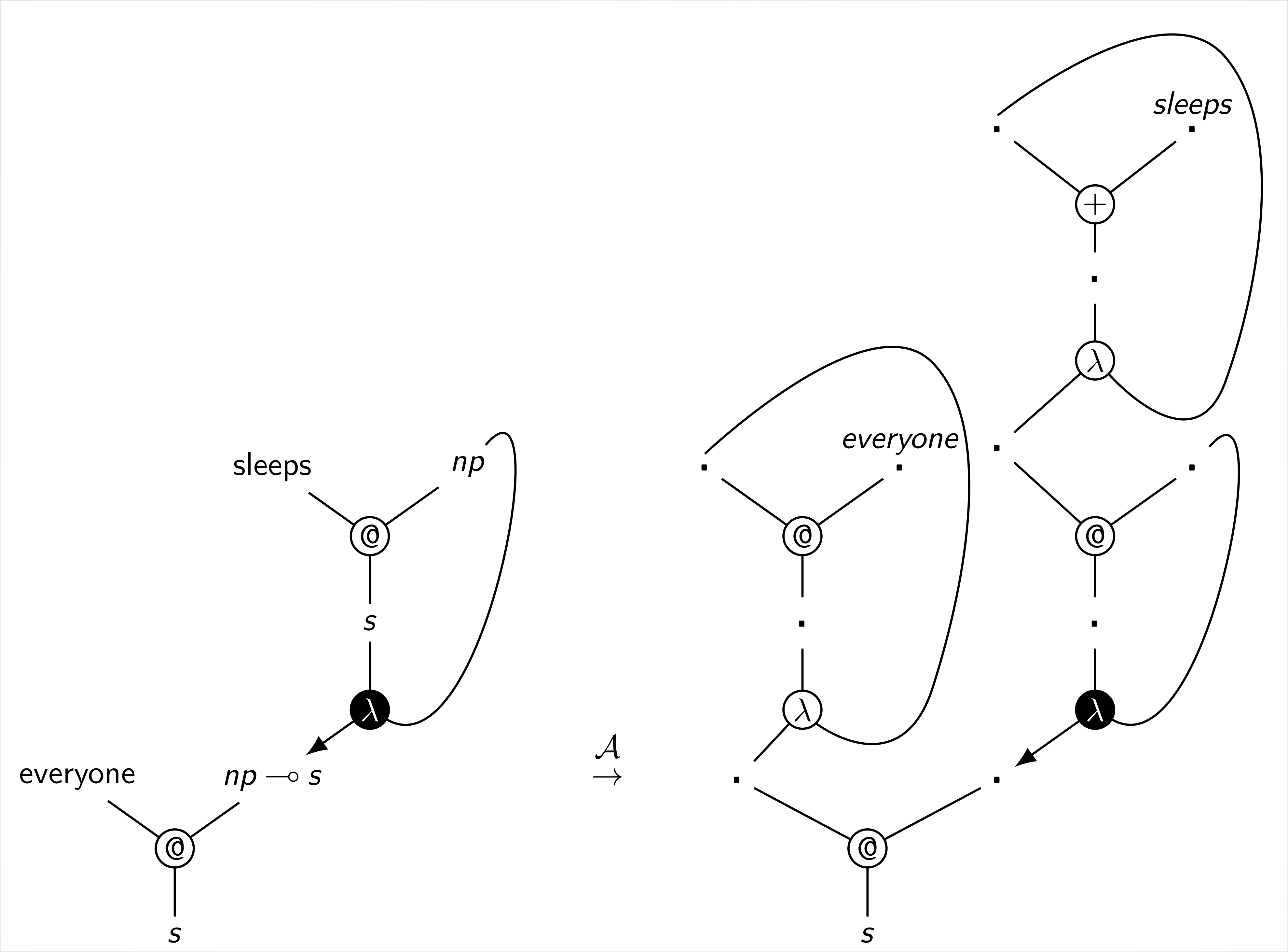


$$Lex(everyone) = \lambda P.(P\ \ everyone) : (np \multimap s) \multimap s$$
$$Lex(sleeps) = \qquad \lambda y.y + sleeps : np \multimap s$$

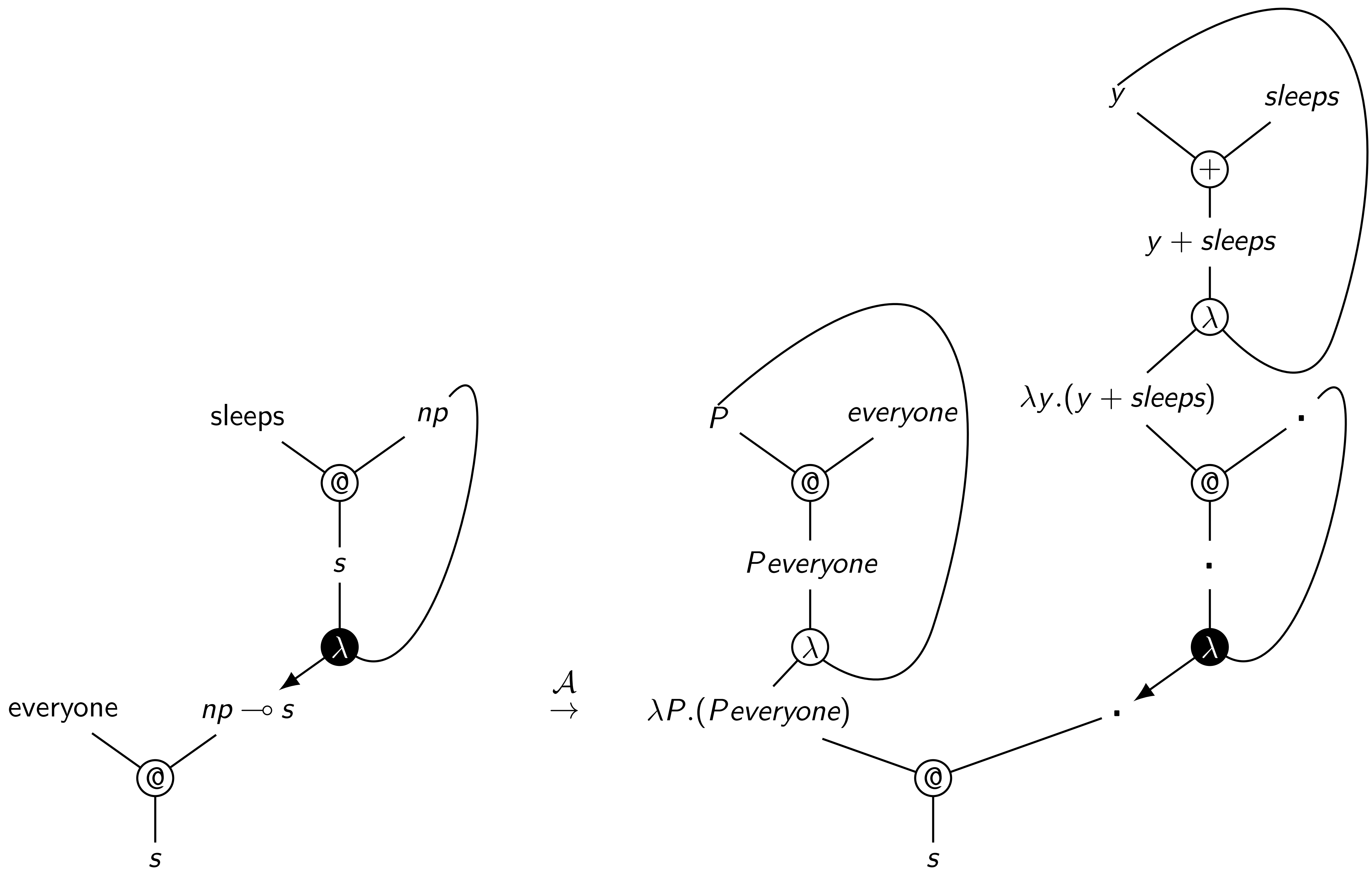
HTLG: EVERYONE SLEEPS



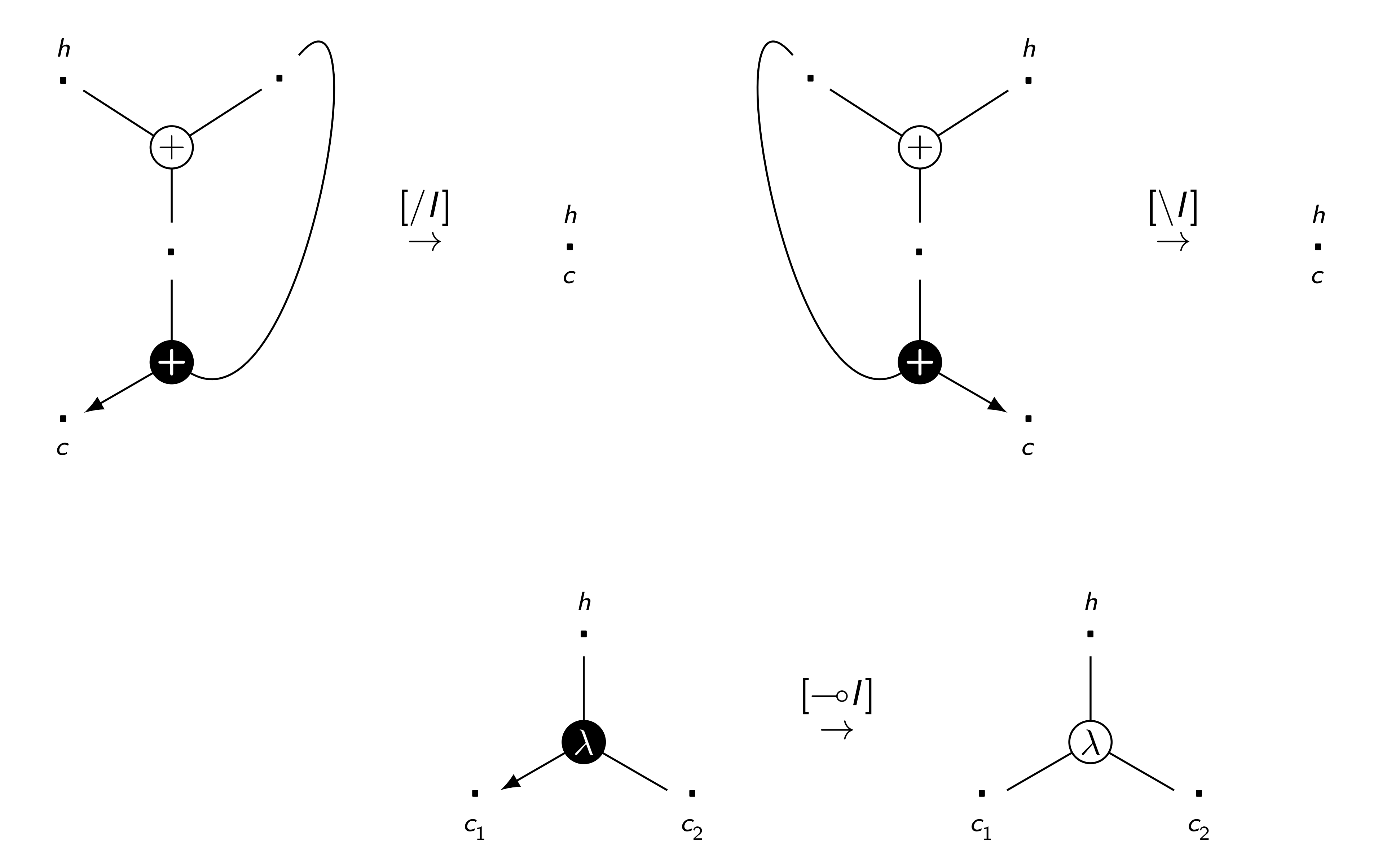
HTLG: EVERYONE SLEEPS



HTLG: EVERYONE SLEEPS

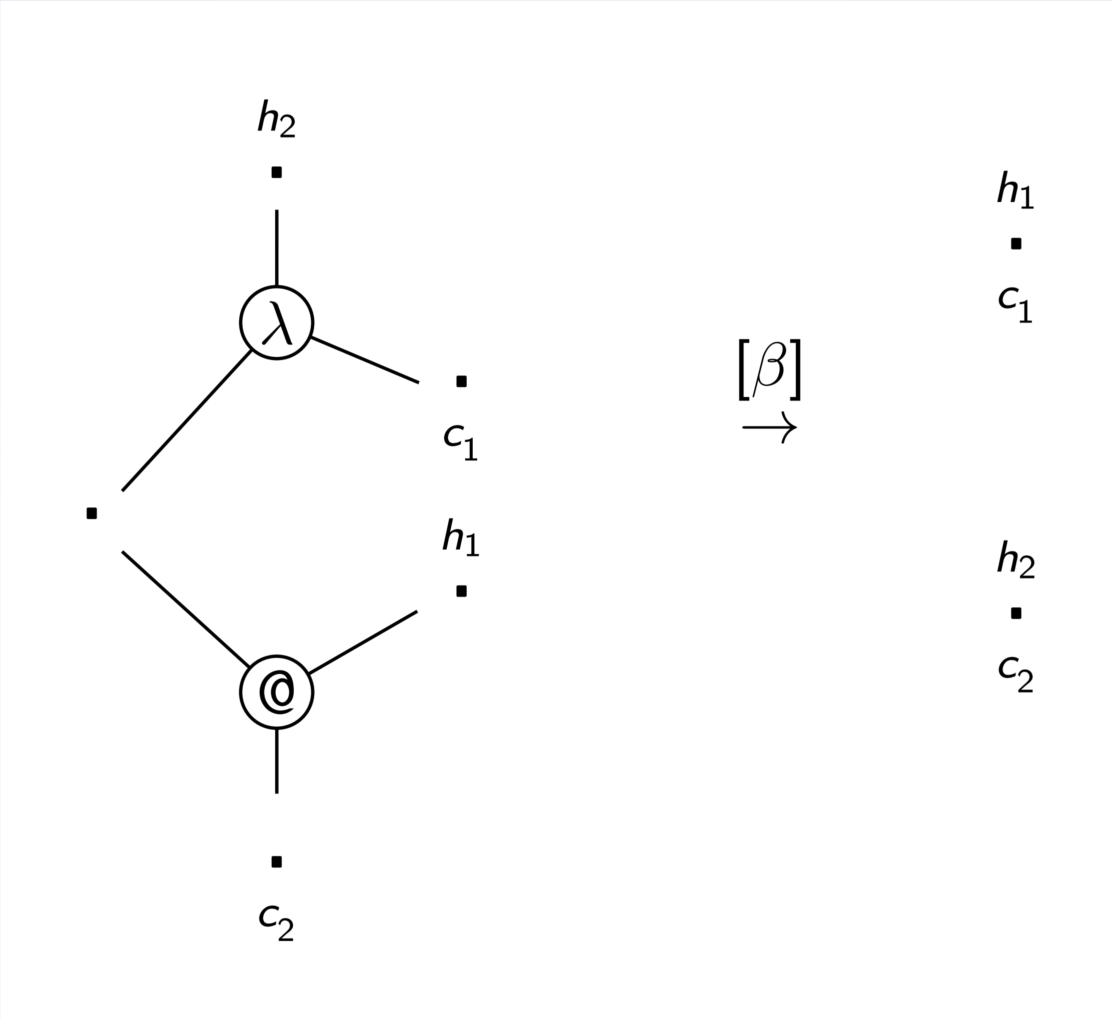


HTLG: CONTRACTIONS



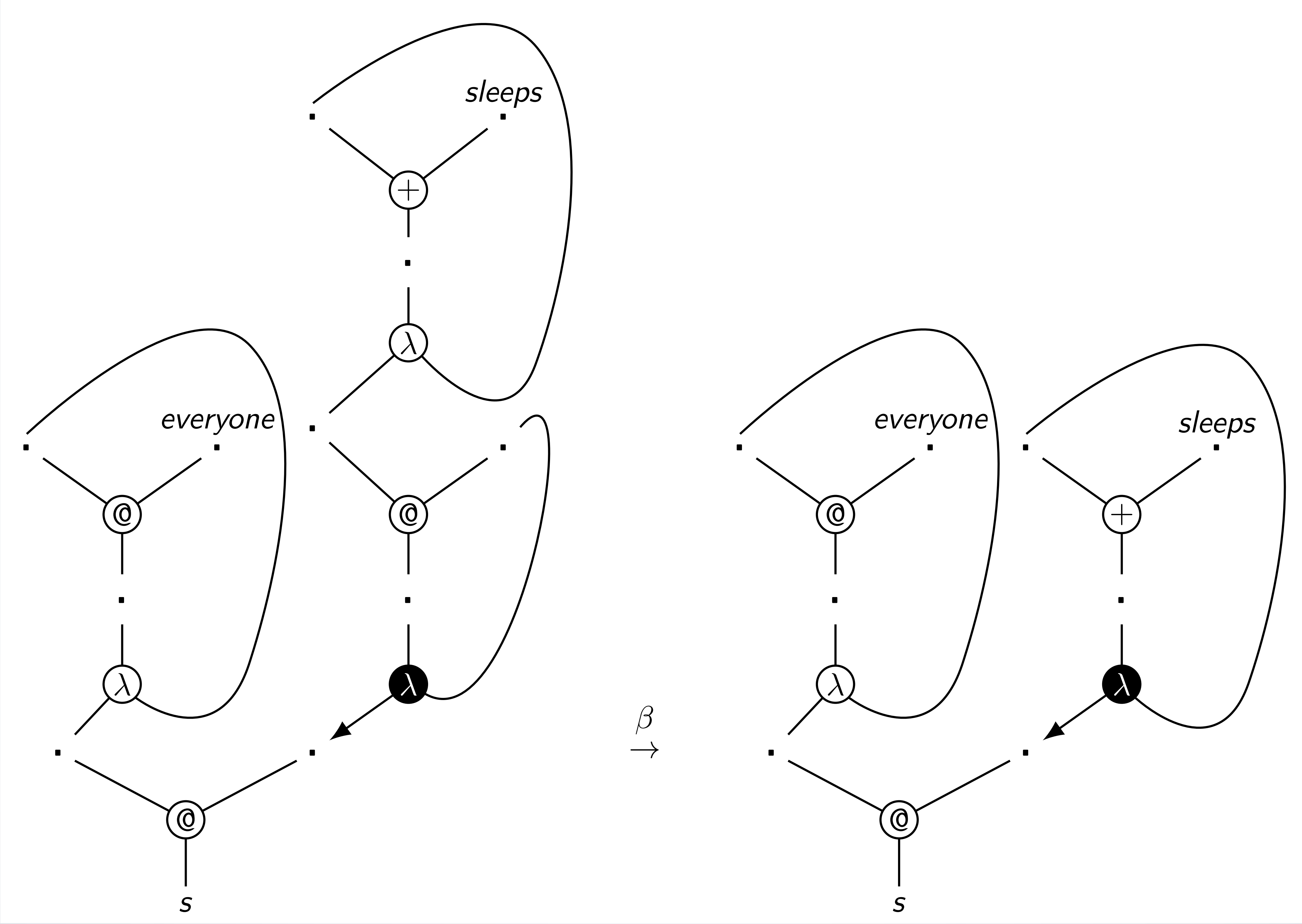
Condition: c_2 must be an ancestor of h by a path which does not pass any asynchronous (par) links

HTLG: BETA RULE

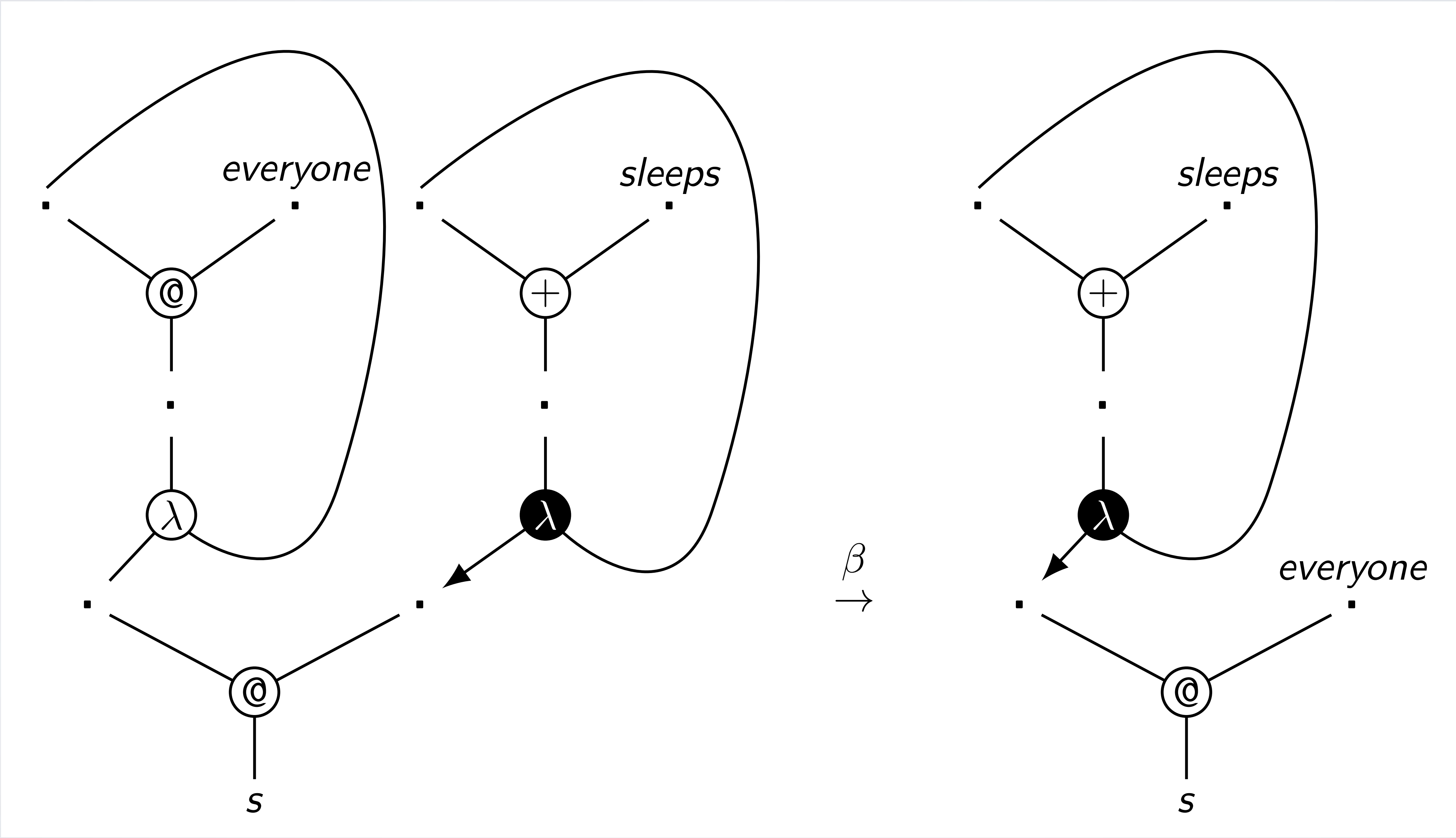


Condition: h_2 must be an ancestor of c_1 by a path which does not pass any asynchronous (par) links

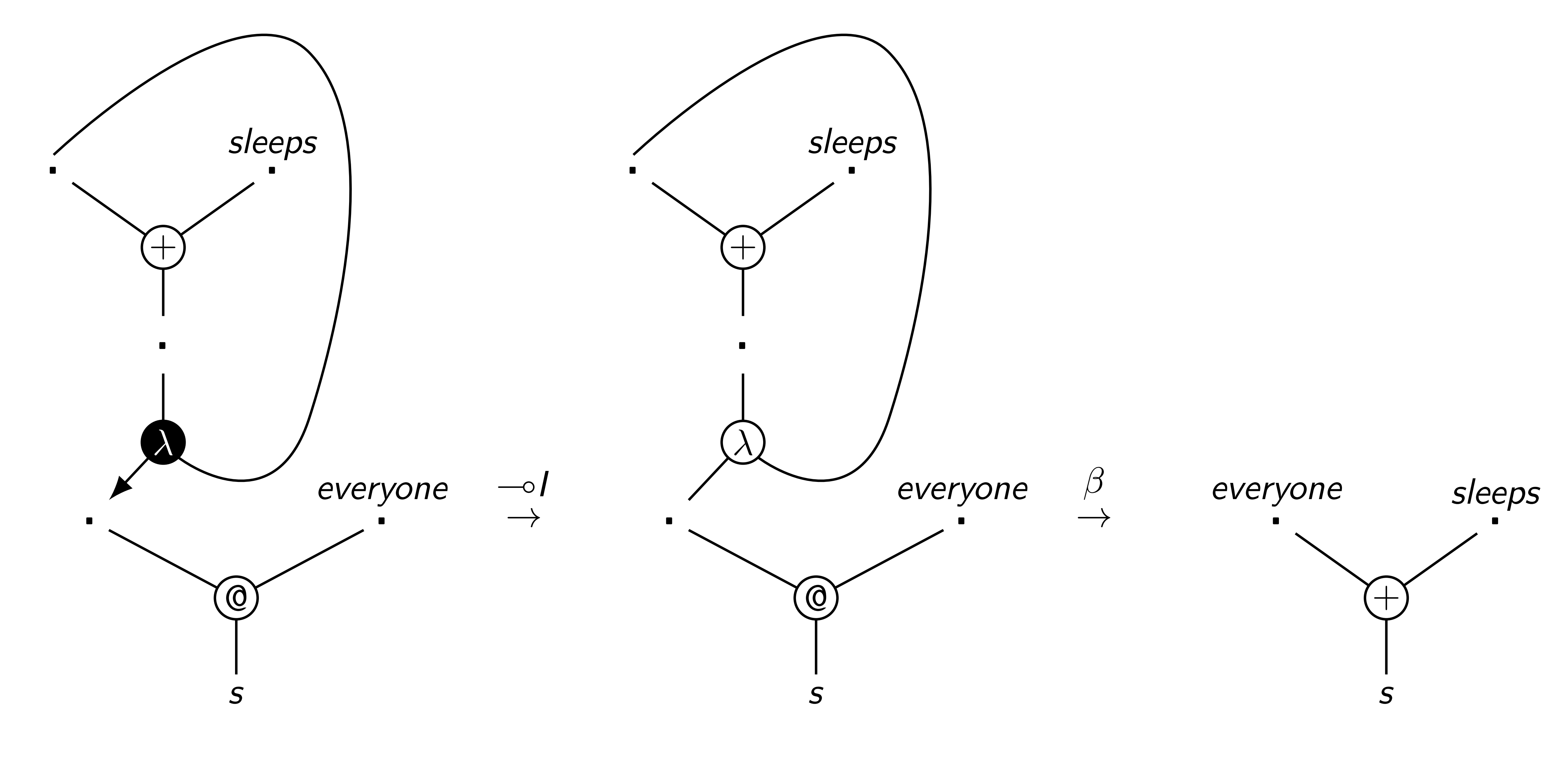
HTLG: EVERYONE SLEEPS



HTLG: EVERYONE SLEEPS



HTLG: EVERYONE SLEEPS

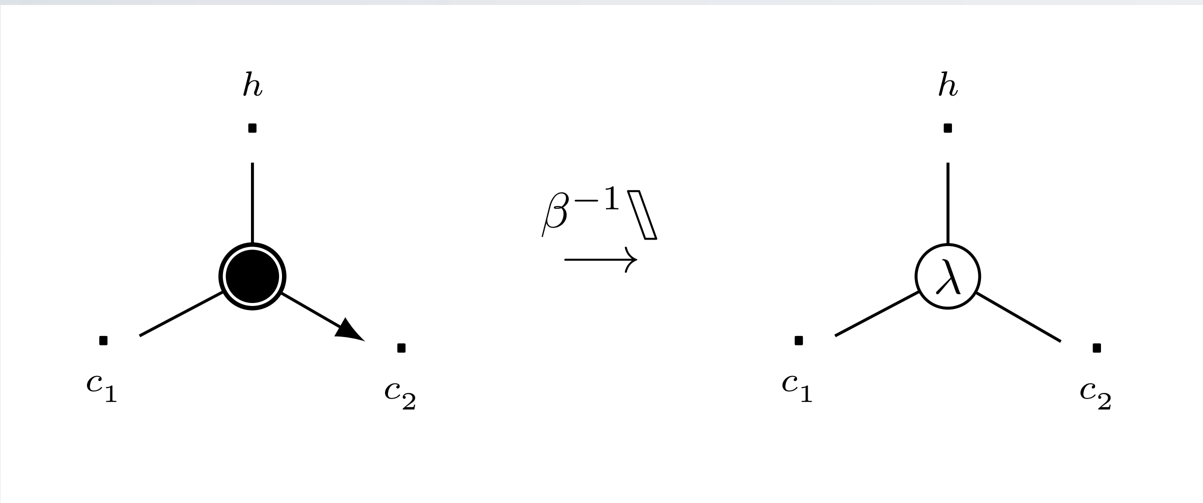


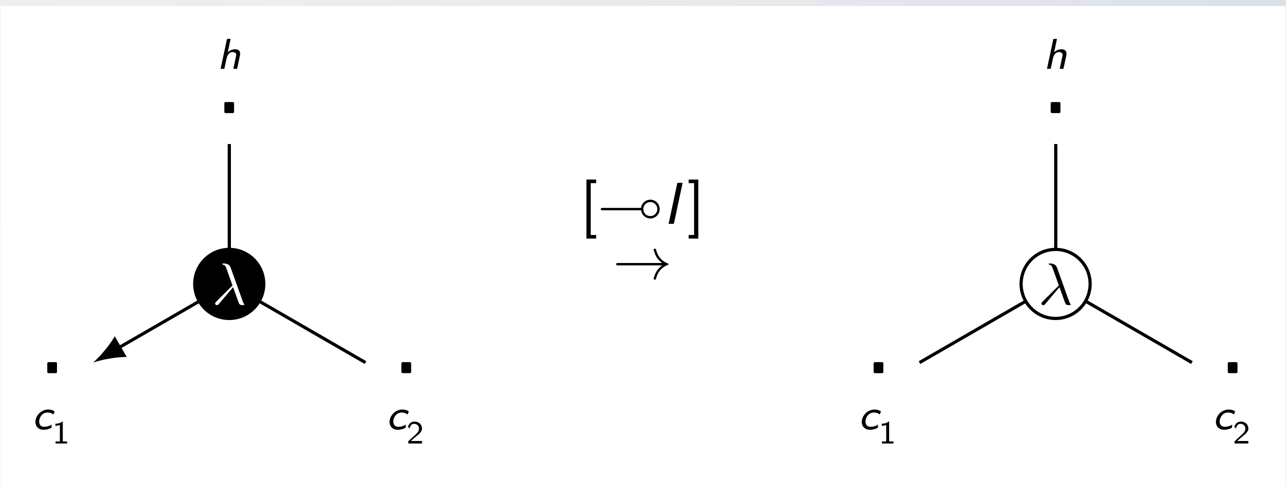
COMPARISONS

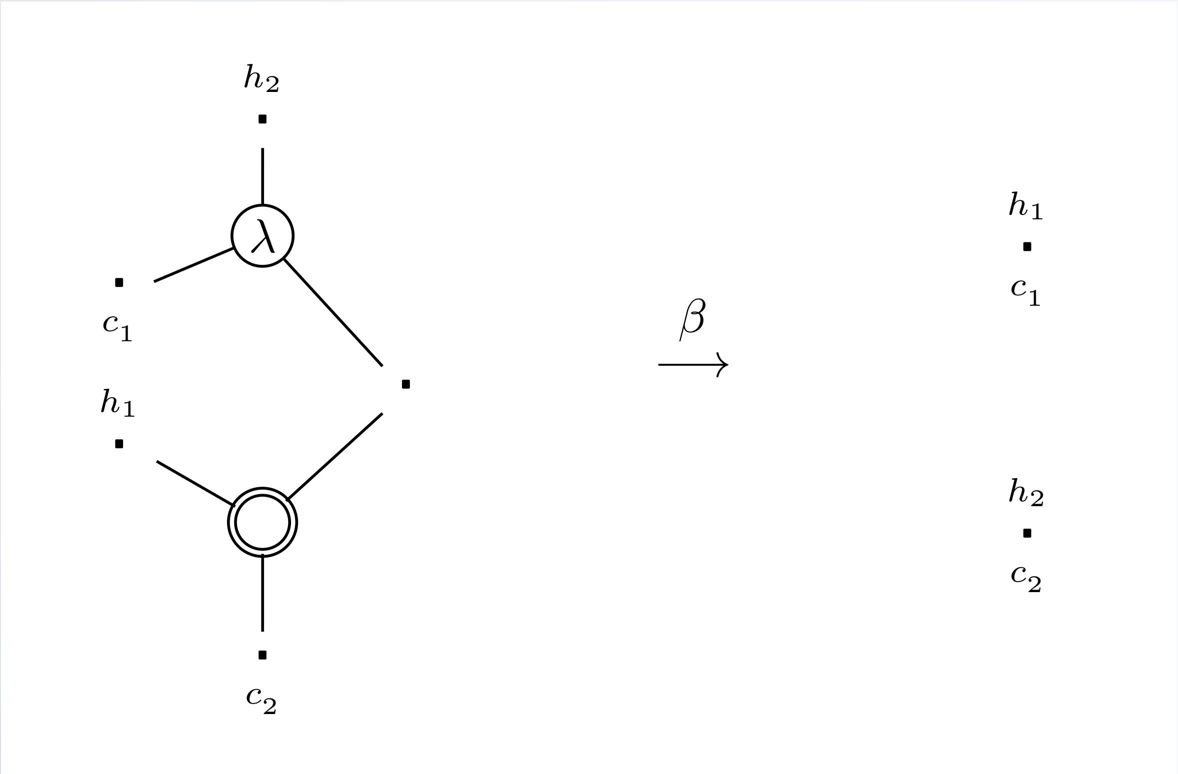
COMPARISON

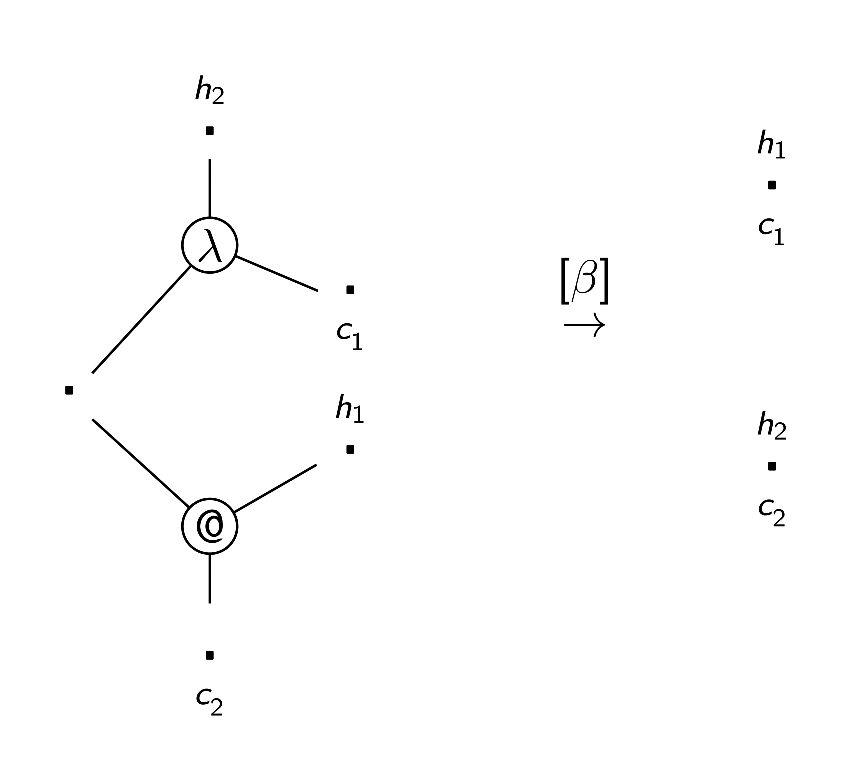
NLλ

HTLG

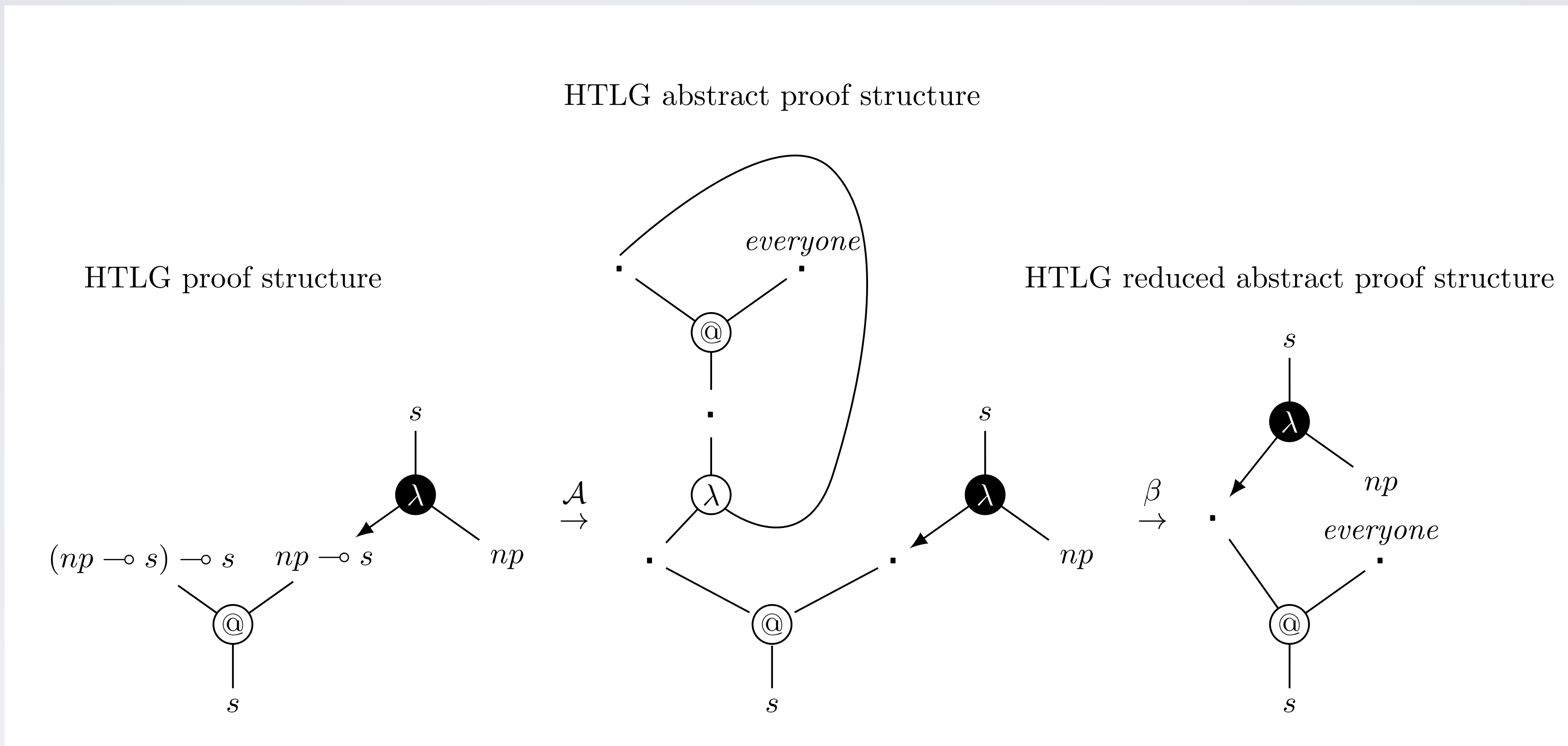


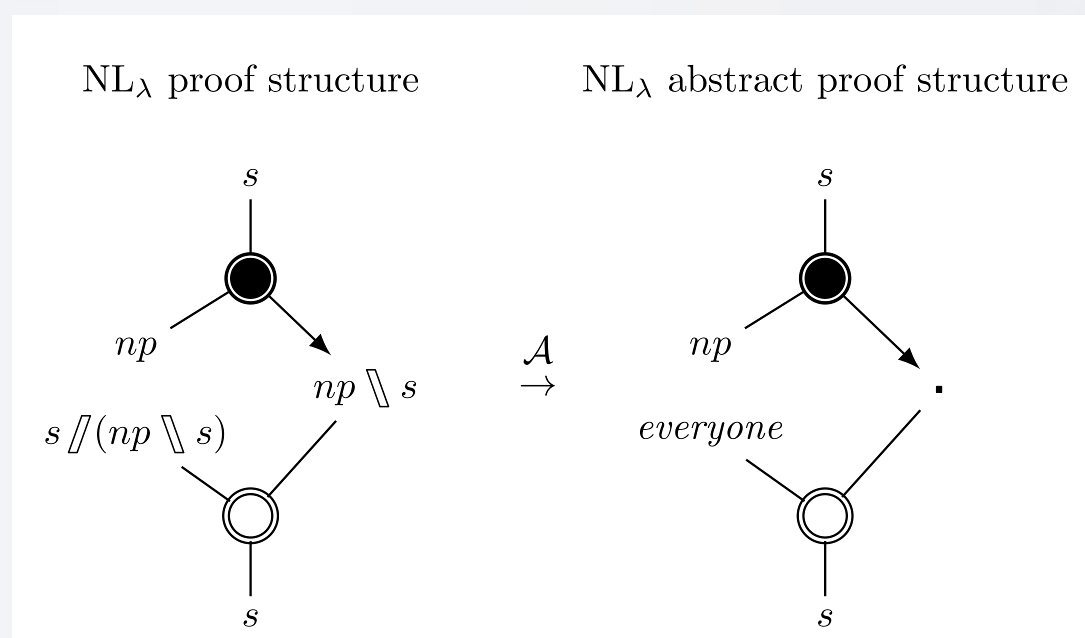






COMPARISON



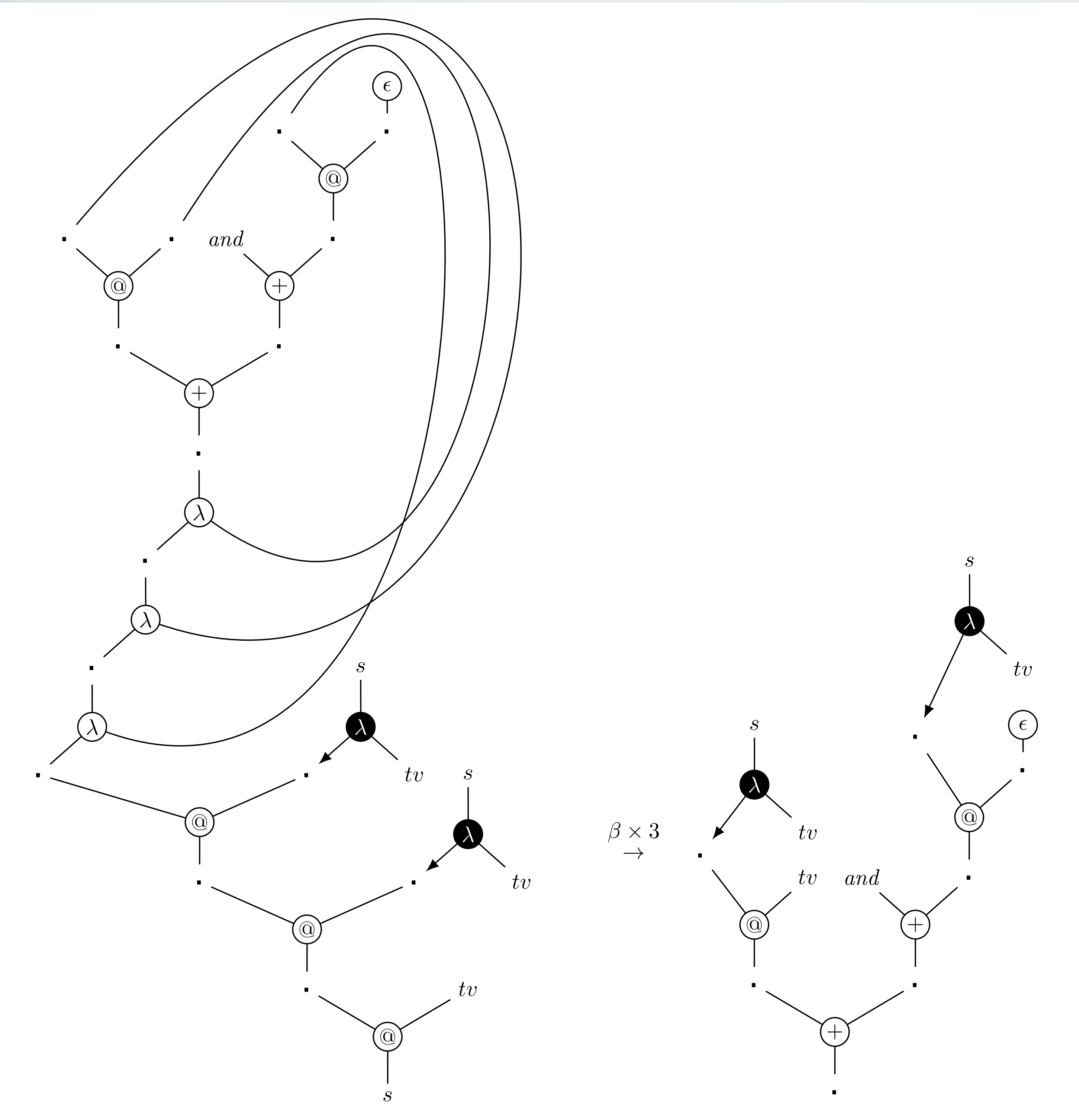


Partial evaluation of redexes in the lexical entry; already used by de Groote & Retoré (1996) and Morrill (1999) for semantics.

TRANSLATIONS

HTLG		NL_λ
$+$ link	$\leftrightarrow$	$\circ$ link
@ with premisses $p_1 - p_2$	$\leftrightarrow$	$\circledcirc$ with premisses $p_2 - p_1$
λ tensor (lexicon)		???
λ par with conclusions $c_1 - c_2$	$\leftrightarrow$	$\bbslash$ par with conclusions $c_2 - c_1$
???		t, $\sslash$, $\odot$ par links
contractions for $/$, $\backslash$	$\leftrightarrow$	contractions for $/$, $\backslash$
???		contraction for $\bullet$
λ par rewrite	$\leftrightarrow$	$\beta^{-1}\bbslash$ rewrite
β rewrite	$\leftrightarrow$	β rewrite
η rewrite	$\leftrightarrow$	contraction for $\bbslash$
???		contractions for t, $\sslash$, $\odot$

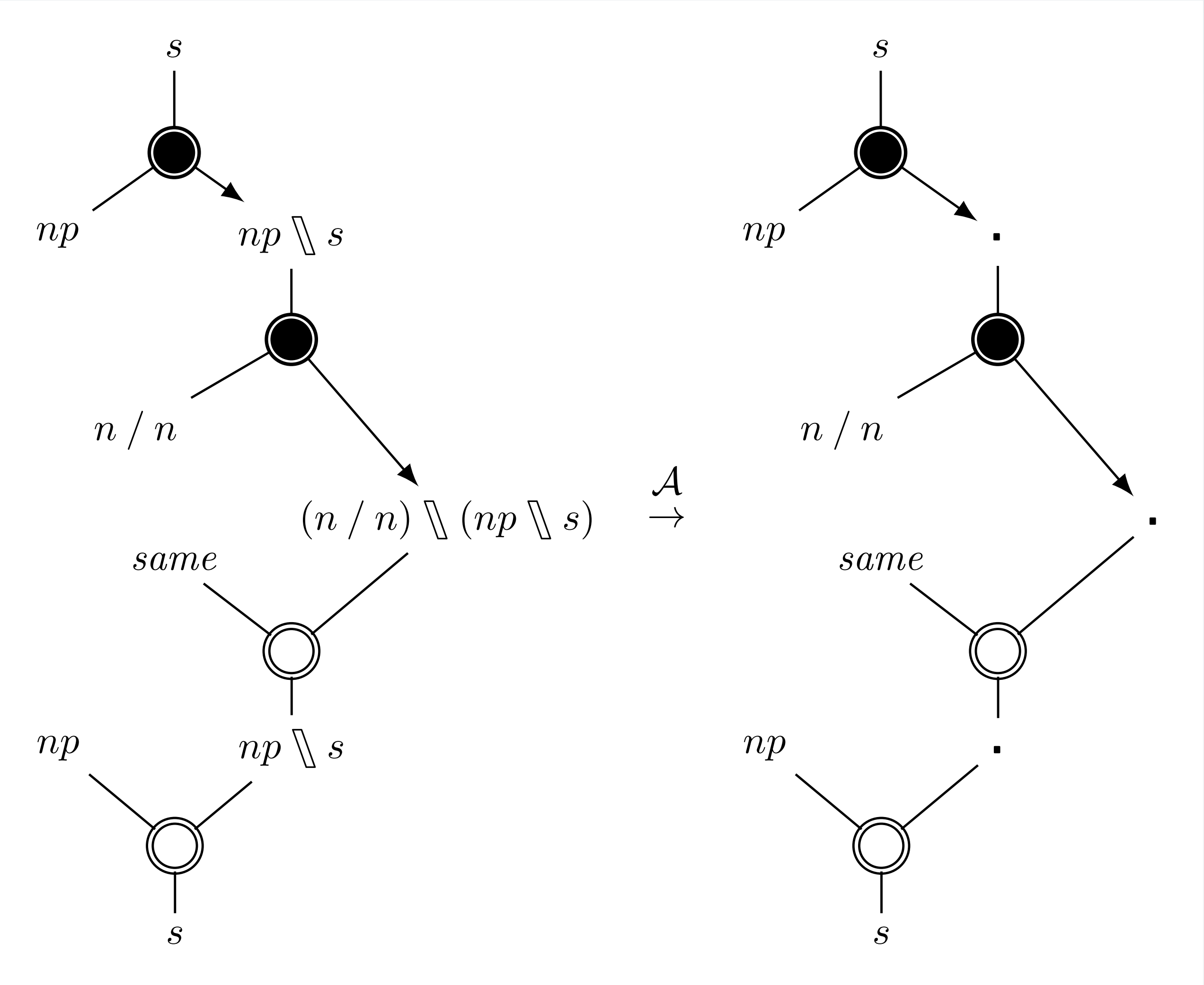
TRANSLATIONS



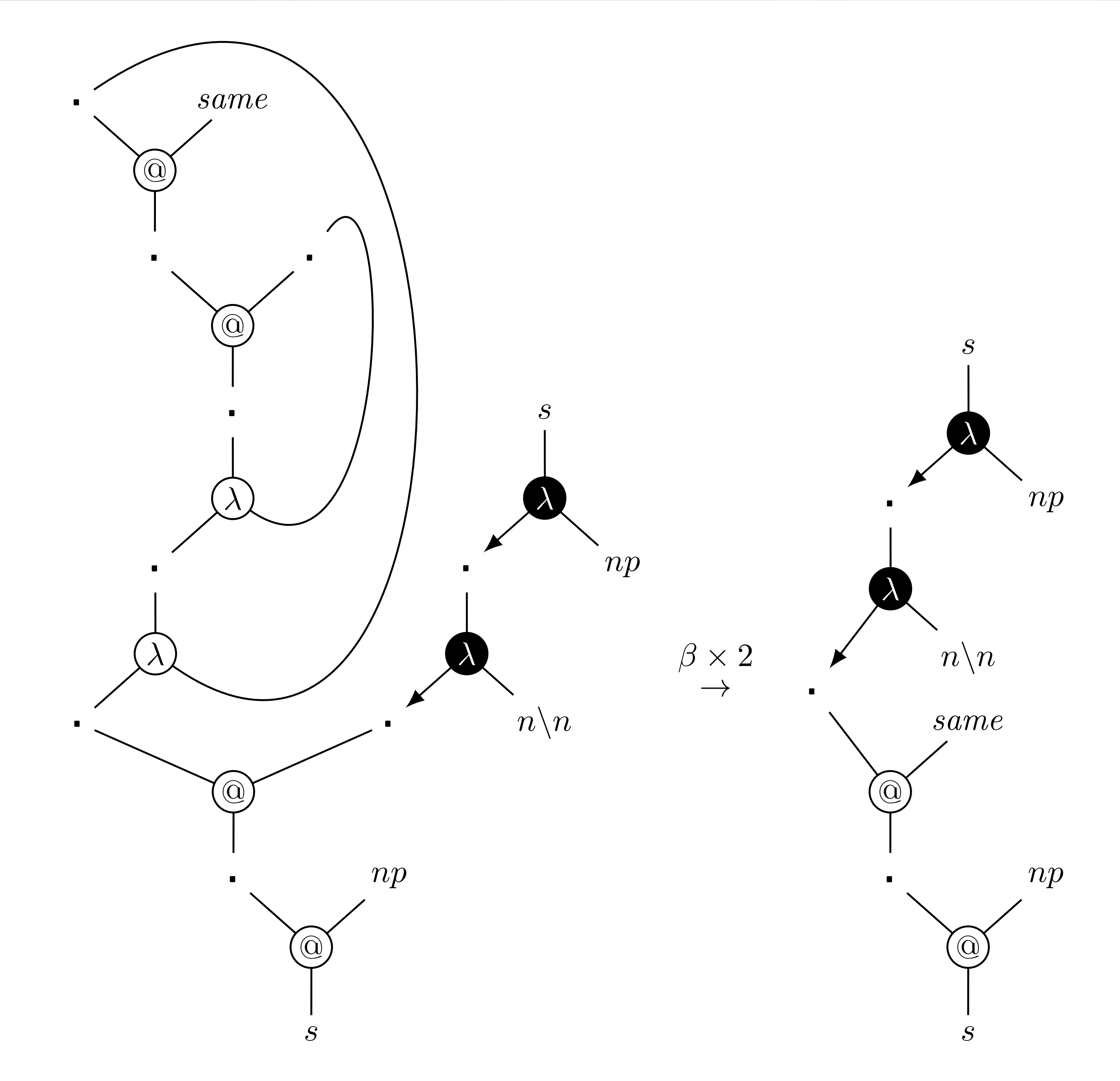
The gapping analysis of Kubota & Levine (2013) translates into NLλ as follows.

$$((tv \odot (tv \mathbin{\backslash\!\!\backslash} s)) \backslash s)/(t \odot (tv \mathbin{\backslash\!\!\backslash} s))$$

TRANSLATIONS



TRANSLATIONS



The analysis of "same/different" from Barker & Shan (2014) translates into HTLG as follows

$$((n\backslash n) \multimap np \multimap s) \multimap np \multimap s$$

$$\lambda P.\lambda x.((P\; same)\; x)$$

TRANSLATIONS

Dutch verb clusters in NL_{λ}

dat	$s_{that} \,/\, s_{sub}$
Jan	np
Henk	np
Marie	np
de	$np \,/\, n$
nijlpaarden	n
zag	$(np \backslash (np \backslash s_{sub})) \mathbin{/\!/} (j \mathbin{\backslash\!\backslash} inf)$
helpen	$j \backslash ((np \backslash inf) \mathbin{/\!/} (j \mathbin{\backslash\!\backslash} inf))$
voeren	$j \backslash (np \backslash inf)$

TRANSLATIONS

Dutch verb clusters in NLλ

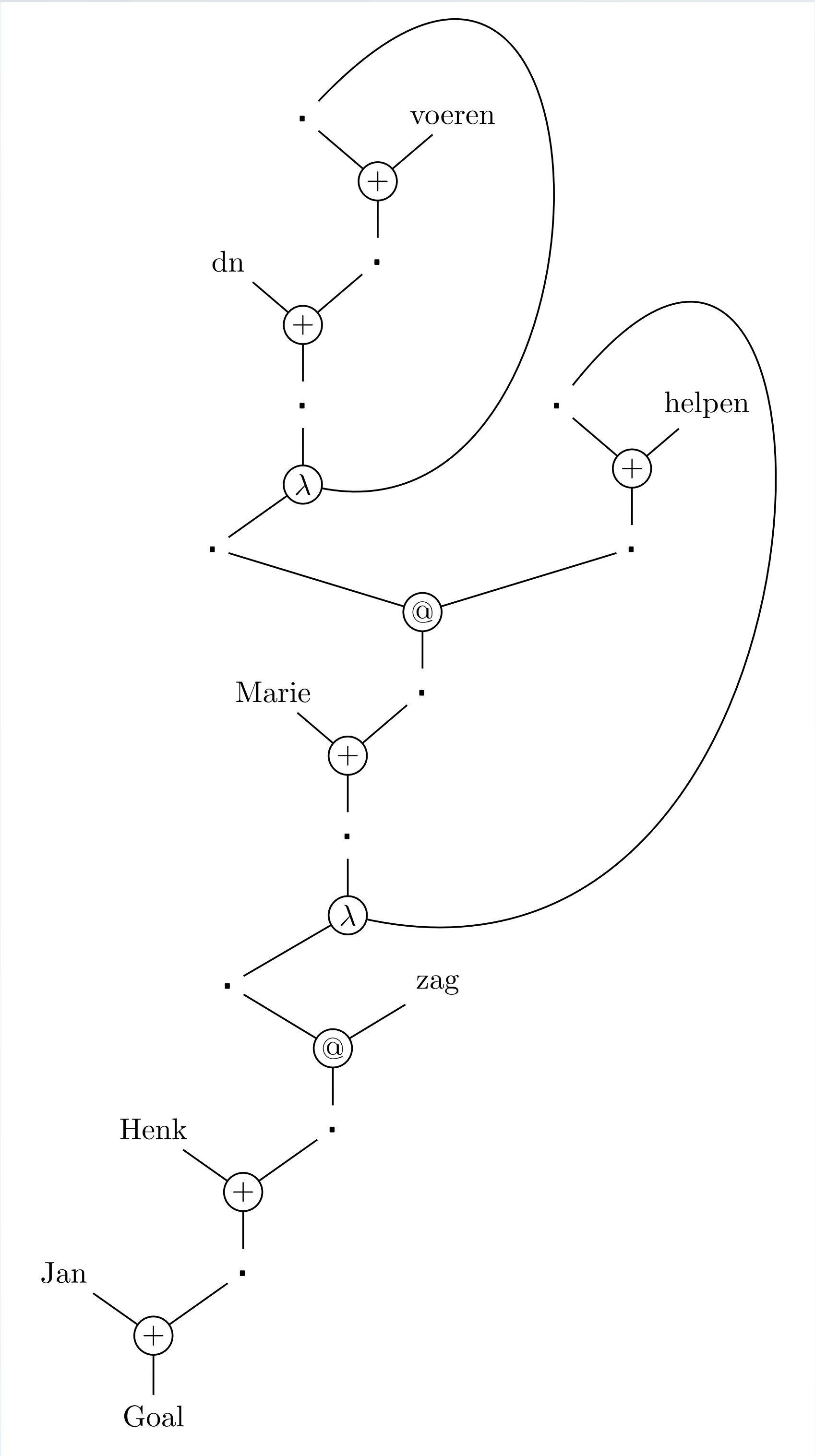
dat	$s_{that} \,/\, s_{sub}$
Jan	np
Henk	np
Marie	np
de	$np \,/\, n$
nijlpaarden	n
zag	$(np \backslash (np \backslash s_{sub})) /\!\!/ (j \backslash\!\!\backslash inf)$
helpen	$j \backslash ((np \backslash inf) /\!\!/ (j \backslash\!\!\backslash inf))$
voeren	$j \backslash (np \backslash inf)$

Compare: Morrill e.a. (2011)

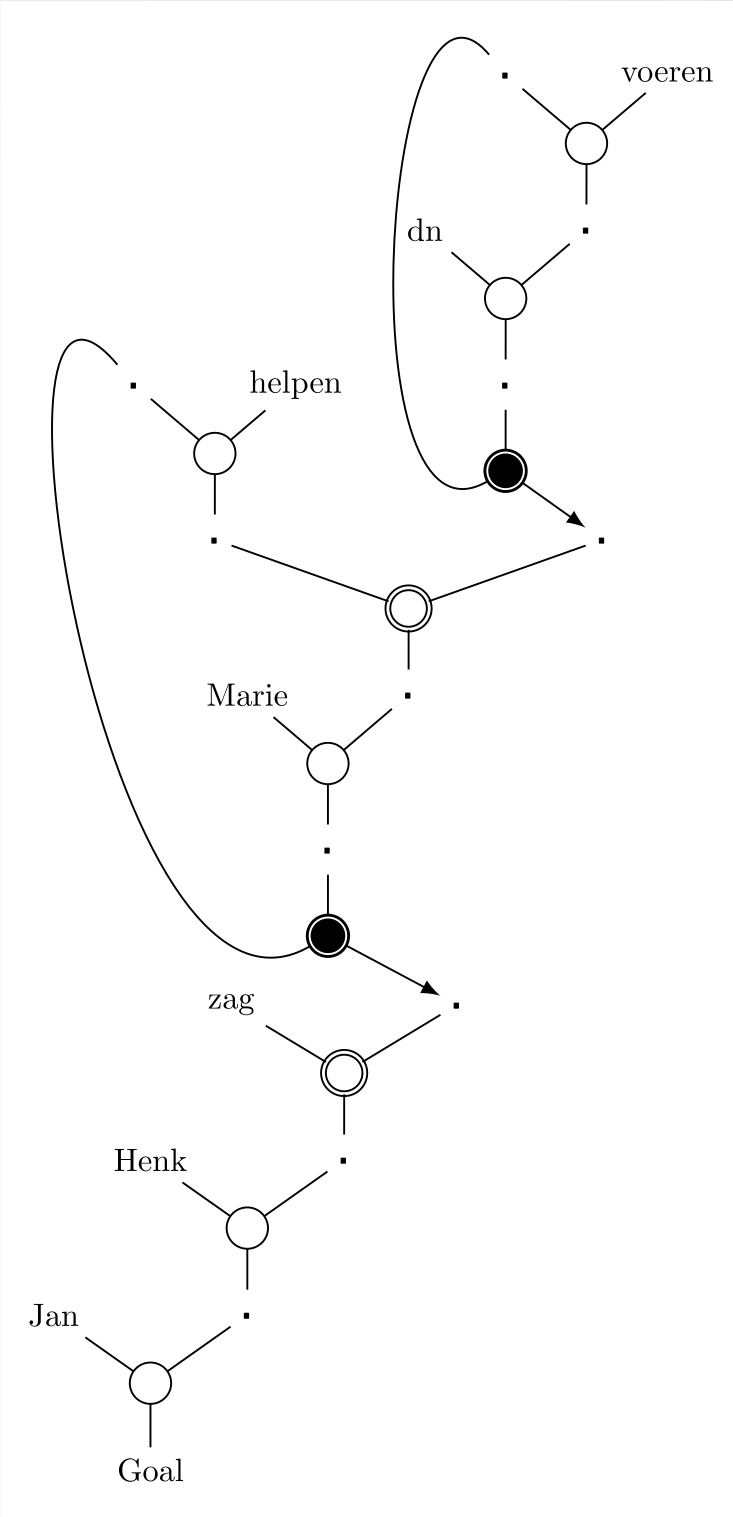
zag	$inf \backslash_w (np \backslash (np \backslash s_{sub})$
helpen	$J \backslash (inf \backslash_w (np \backslash inf))$
voeren	$J \backslash (np \backslash inf)$

TRANSLATIONS

HTLG



NLλ



CONCLUSIONS

- Despite starting with different primitives, HTLG and NLλ produce structures which are related by a simple isomorphism for many of their key linguistic analyses.

CONCLUSIONS

- There appears to be a “common core” of phenomena which can be handled by most type-logical grammars.

- Differences around the edges: higher-order lambda terms allow expressivity which appears to be out of reach for the Displacement calculus; the Displacement calculus can refer to the linear order of gaps.

CONCLUSION

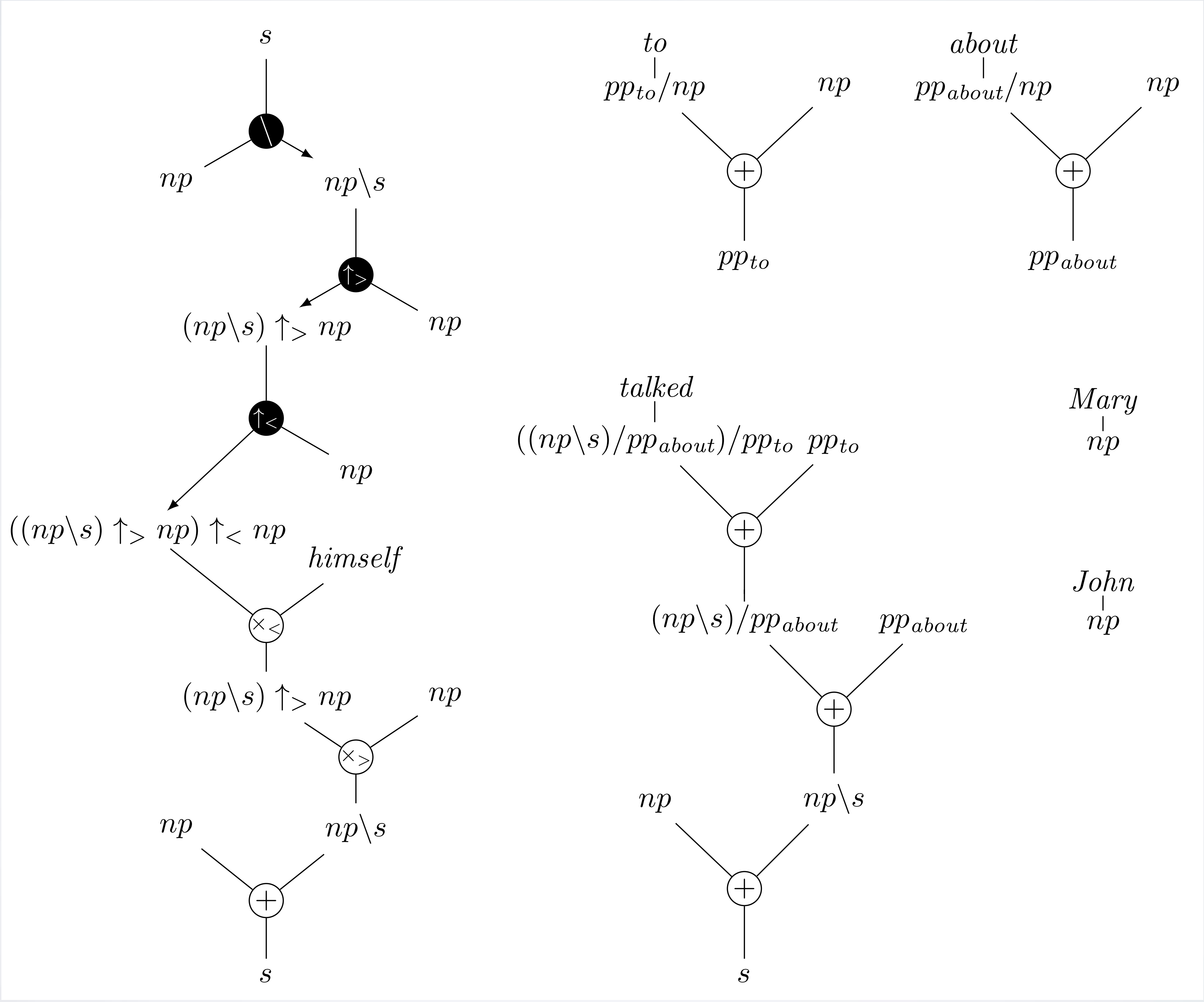
- Single overarching proof theory for monder type-logical grammars
- We can add different “packages”: associativity, beta reduction, wrap
- Makes correspondence between many analyses in different formalisms clear

FUTURE WORK

- Implementation of the graph based formalism in its full generality (using existing graph rewrite tools)
- Beyond the multiplicative fragment?
- More precise relations between different logics and grammars
- Formal language theory?

THANK YOU!

MARY TALKED TO JOHN ABOUT HIMSELF



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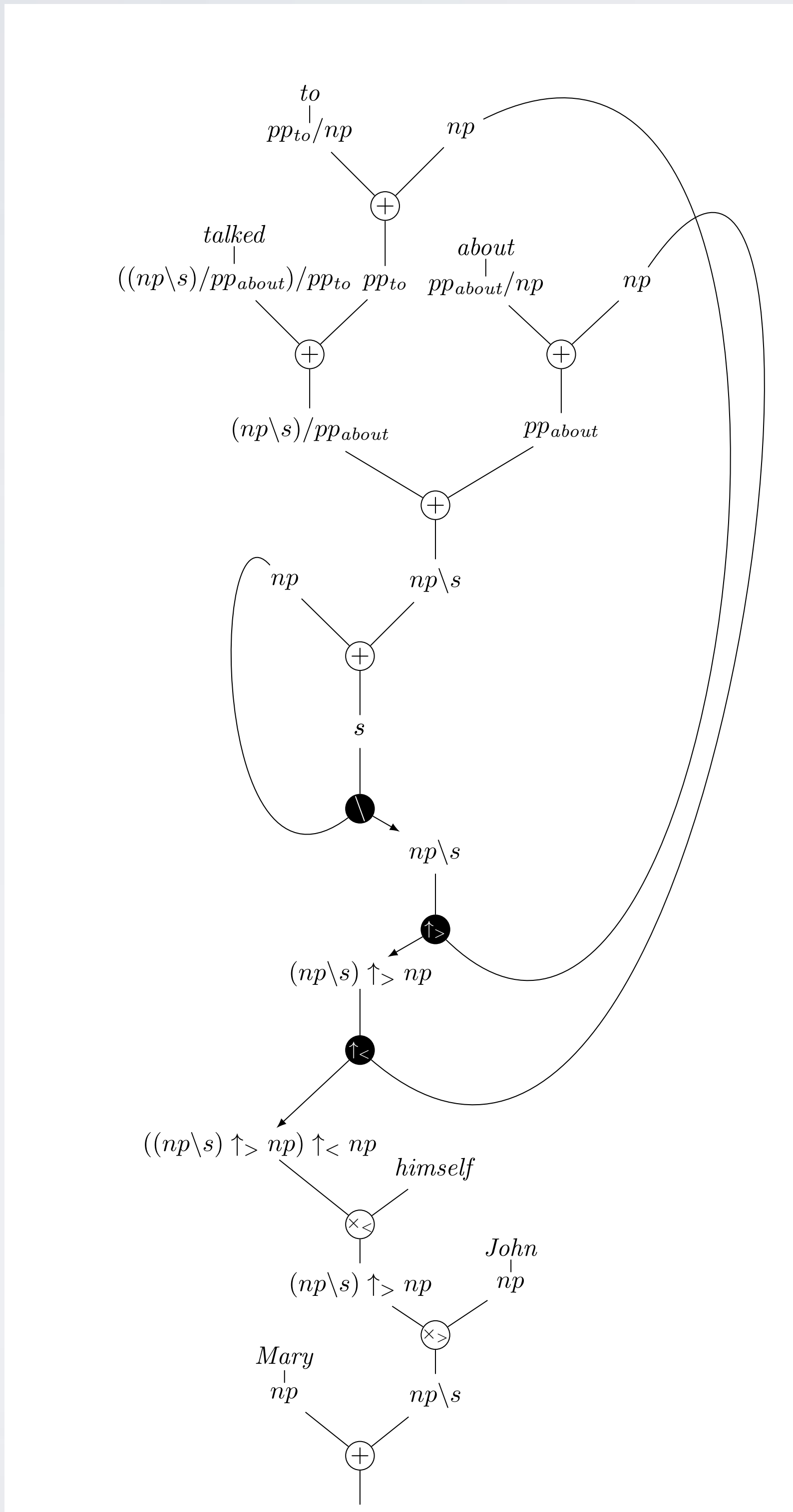

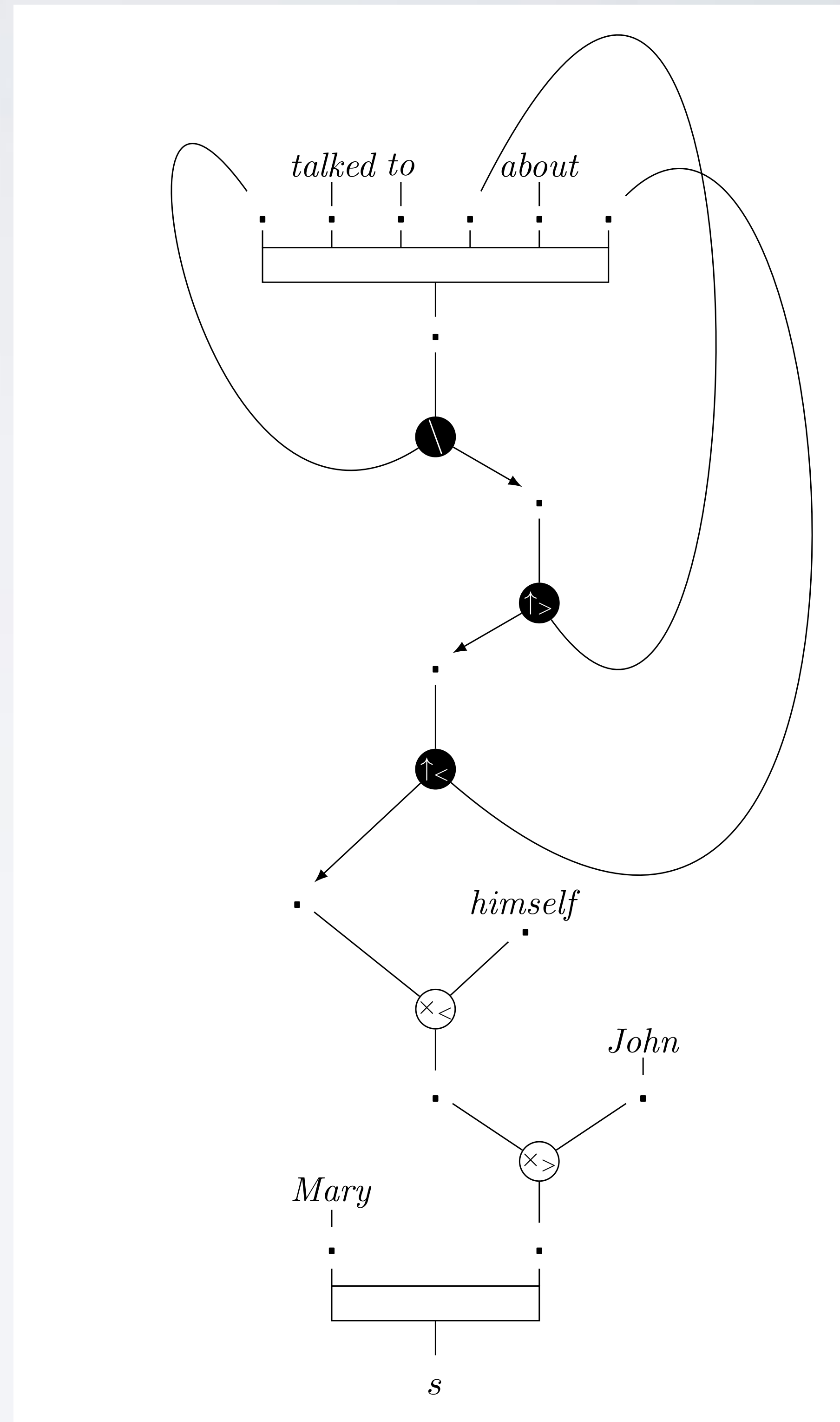

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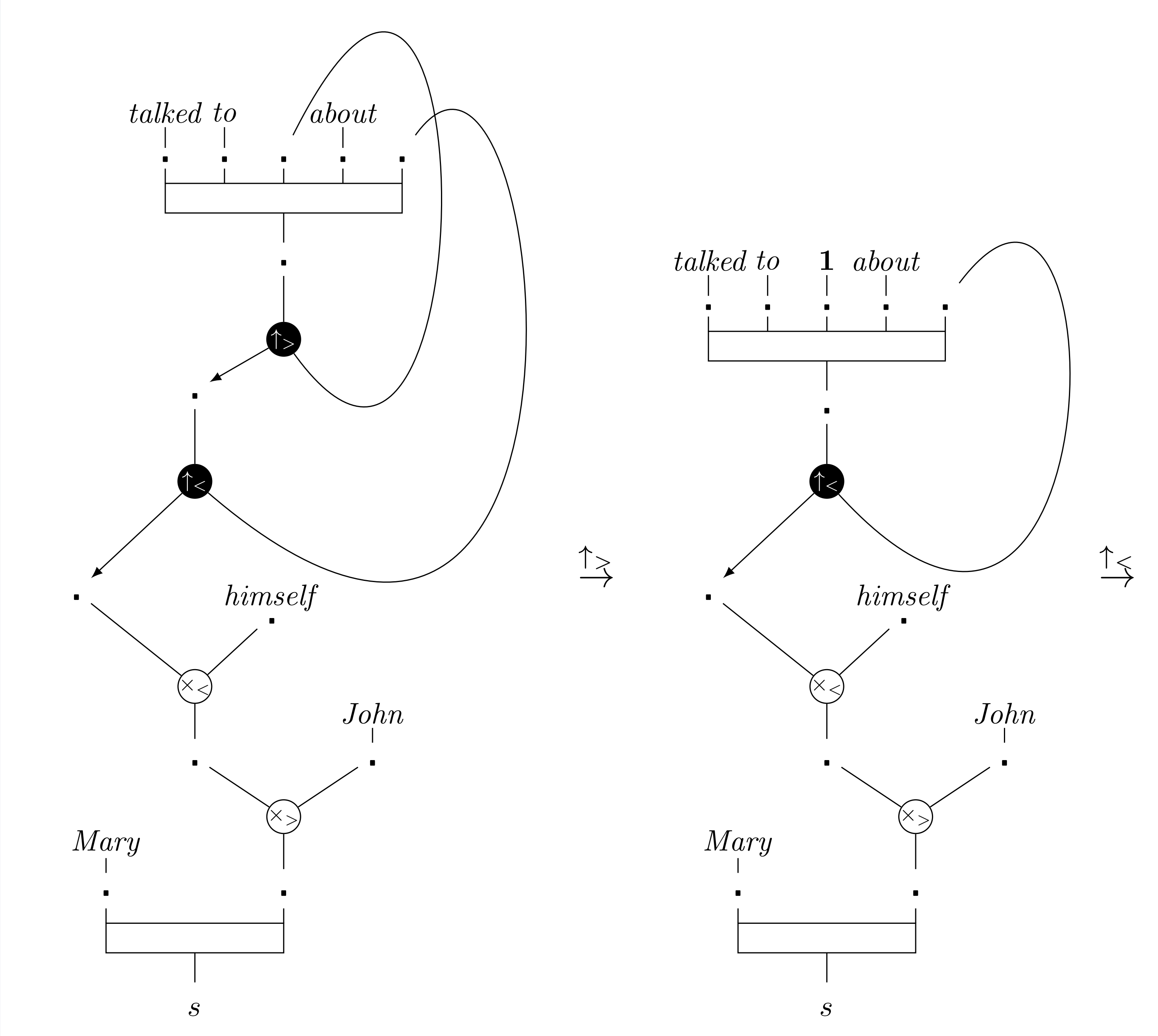

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